



Research Article /Review Article:

Determining Sphere Arc Length for 4D Routing Problem

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Abstract

The concept of dimension is prominent in the researches of physics and mathematics and is also a subject of research in many sciences. While it is mentioned in many theories that the universe is multidimensional or even infinite, in this thesis study, the concept of dimension is addressed up to the fourth dimension. While the fourth dimension is examined in particular, the concepts of latitude, longitude, altitude and time are mentioned, and the concept of 4D navigation with the fourth dimension allows for the accurate prediction of the orbits defined in both time and space, and since it comes to the forefront more in the aviation field, its developments in this sector are mentioned. In the thesis study, it is aimed to determine the most suitable and closest flight path by performing route analysis with geodetic methods in 4-dimensional orbit determination. Thus, the application of the 4D orbit determination method will be investigated by SESAR and NextGen and will allow air traffic controllers to accurately predict the effect of a disturbance in air traffic flow in the near future. In addition, the closest distance calculated with different approaches of flight routes was compared on the sphere and ellipsoid, and 7 flight routes were examined by taking Samsun as the center. In addition, the route length was calculated from the sum of the distances between the intermediate points forming the flight route.

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Contribution to the field statement:

This study compares six spherical radius models for calculating flight-route distances between airports in Türkiye. By applying spherical trigonometry to three routes originating from Samsun, it quantifies differences in calculated distances and provides a basis for evaluating spherical models in route planning for four-dimensional air navigation.

Highlights:

- Six spherical radius models are compared for flight-route calculations.
- Three routes from Samsun are evaluated using spherical trigonometry.
- Calculated distances vary according to the selected sphere radius.
- The findings support further evaluation of distance models for air navigation.

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1. INTRODUCTION

4D orbit determination system is prominent in maritime and aviation fields. Since the length of travel time is important in the trajectory planning stage in maritime, fuel requirement is taken into account and fuel planning is needed. For this reason, 4D orbit determination and prediction processes are mostly used in the aviation field (Lucas, 2017).

The update and major transformation of Air Traffic Management (ATM) systems used today is extremely important because air traffic is constantly growing. The basis of this transformation is the transition from the

current clearing- based Air Traffic Control (ATC) operations to orbit- based operations. Due to external factors such as airspace congestion, weather conditions, temporary military activities, airspace restrictions, etc., the actual flight process of aircraft sometimes does not follow the planned route due to disruptions. This situation reduces the predictability of air traffic, increases the difficulty of air traffic management, and prevents the efficient operation of the air traffic management system. In order to increase the robustness of the 4D conflict- free orbit, these disruptions must be taken into account in the orbit planning process.

Therefore, the four- dimensional orbit of the aircraft must be estimated (Lucas,2017).

The four- dimensional (including longitude, latitude, altitude and time) trajectory prediction of aircraft is a key technology for current automation systems, as well as the basis for future trajectory- based operations. The implementation of 4- dimensional trajectories (4DTs), which are trajectories defined in both time and space, will allow air traffic controllers to accurately predict the impact of a disturbance to air traffic flow in the near future. Therefore, the ability to predict aircraft trajectories becomes very important to properly optimize the increasingly complex ATM (Air Traffic Management) systems used today (Lucas, 2017).

Different approaches are used in the discipline of cartography to find the most suitable and shortest flight routes used in air navigation. As a result of the comparative analysis of these approaches, the most suitable method was determined with the calculations made on spherical and ellipsoidal solutions and detailed mathematical formulations of solution methods in order to find the shortest distance in terms of flight route length in air navigation.

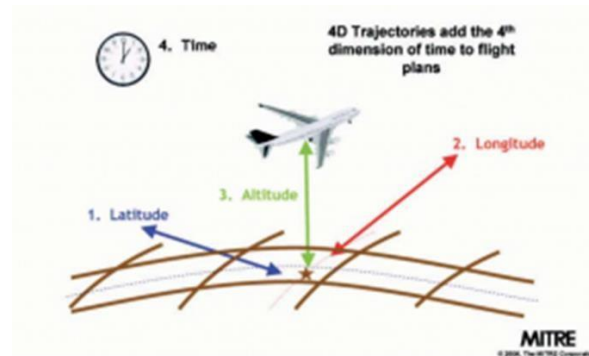


Figure 1. 4D Trajectory add the 4th dimension of time to flight plans

The process of finding the shortest distance is carried out in two stages. The first stage is the operations performed on the sphere and was determined with two methods. These methods are; the methods of finding the shortest distance on the orthodromic curve with different sphere radii and authentic sphere. The orthodromic curve on the sphere is called the largest circle arc in spherical trigonometry. The shortest length between two points is calculated using the edge cosine relation on the orthodromic curve. The second stage is the operations performed on the ellipsoid and in order to determine the most suitable path in terms of distance, the sphere coordinates (φ , λ) that we have for the ellipsoidal solution were converted to ellipsoidal coordinates (B , L) and the ellipsoidal operations were started. The aim of this study is to determine the most suitable route, which is one of the titles within the scope of 4D trajectory. In this process, spheres were used instead of ellipsoids and the arc length was calculated using spherical trigonometry.

2. MATERIAL AND METHODS

2.1. Methods

2.1.1. 4D Routing

The Federal Aviation Administration (FAA)/ Eurocontrol Action Plan 16 (AP16) defines trajectory as a four-dimensional (latitude, longitude, altitude, and time) description of an aircraft's flight path (FAA, 2020). The International Civil Aviation Organization (ICAO) has expanded the definition of trajectory to include ground operations and describes the aircraft's motion in the air and on the ground, including position, time, speed, acceleration, and other attributes (ICAO, 2020). The organizations' different definitions of trajectory are expanding and changing day by day because the global air transportation system is the cornerstone of the world economy and because of developing technology.

Trajectory prediction is the process of predicting future aircraft states based on current aircraft states, estimates



of pilot and controller intent, expected environmental conditions, and computer models of aircraft performance and procedures. Four- dimensional aircraft trajectory prediction is a key technology for

The benefits of 4D orbital operations include: improved air traffic operations by increasing the overall predictability of traffic, optimum operations for airlines (aircraft using preferred routes and levels), better service provision (due to ground- ground and air- ground interoperability), reduced costs (e.g. fuel and/ or time), reduced emissions, increased capacities (en route and airport), controllers will be able to handle more traffic safely, and it will be easier for controllers to manage traffic (less conflict, information arrives much in advance).

Currently the implementation of 4D orbit management is being investigated by NextGen in the US and SESAR (Single European Sky ATM Research) in the EU.

2.1.1.1. NextGen

The Next Generation Transportation System (Next Generation) program was designed to be phased in three time periods: Research and Development activities (2007-2011), aircraft equipment and deployment capabilities (2012-2018), and a fully integrated all-weather ATM (Air Traffic Management) system for the entire air transportation field (2019-2025). The main objective of this project is to renew the ATM in use today to meet the growing needs of air traffic users.

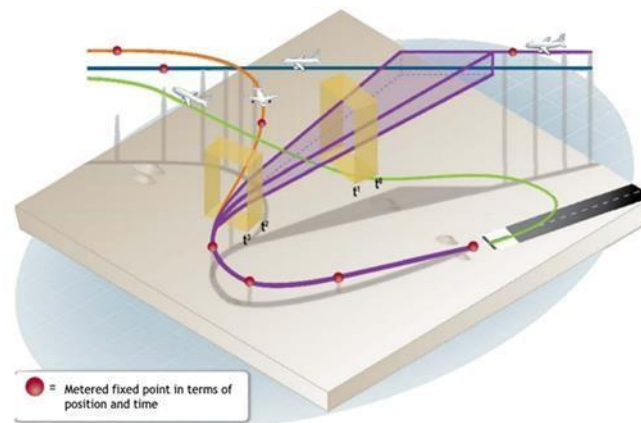


Figure 2. NextGen 4D orbit definition diagram

The four main elements of the Next Generation program are:

1. ADS- B application Ads- b is a system that allows its users to broadcast very accurate and diverse information using GPS (Global Positioning System), the most important of which is the current aircraft position.
2. Next Generation Data Communications Although most of the communication between air traffic controllers and aircraft crews today is done by voice, it would be valuable to have information transmitted by other means (and in a faster, more automated manner) to enable this communication. The aim is to enable controllers to handle a greater volume of traffic through data communication.
3. Next Generation Network- Enabled Weather (NNEW)- The main goal of NNEW is to combine thousands of different weather observation sources and ground sensors and combine them into a single, concise, real- time picture available to all air traffic users.
4. National Airspace System Voice Switch (NVS) - The purpose of the NVS is to consolidate the many different voice systems used in the National Airspace System (NAS) today into a single set of scalable air/ ground and ground/ ground voice communications switches.

2.1.1.2. SESAR

The SES (Single European Sky) programme was launched by the European Commission in 2004 with the aim of reforming European airspace. The main objectives of SES are:

1. Restructuring of European airspace as a function of air traffic flows.



2. Creating additional capacity.

3. To increase the overall efficiency of the ATM system.

The goals and vision of SESAR are largely based on the concept of orbit- based operations and the provision of air navigation services that allow aircraft to fly in their preferred orbit without many external constraints

Another important ideology of SESAR is that to properly improve ATM performance, current systems must start viewing flights as a whole in a flow and network context, rather than in fragmented segments as they are today. There must be improvements at every stage of the flight, as shown in the image below (Figure 3).

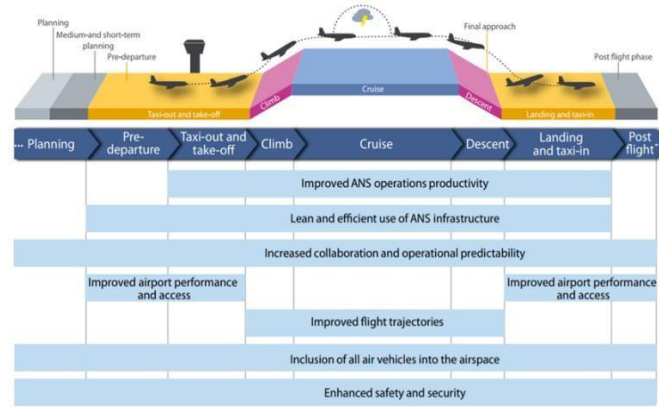


Figure 3. Target improvements for each flight phase of SESAR

2.1.1.3. Route Determination with Spherical Triangle

A spherical triangle is a geometric structure formed by connecting three points on the surface of a sphere with great circle arcs. William Marshall Smart (1977) defined the spherical triangle as one of the basic tools of spherical astronomy and stated that these triangles are indispensable for determining the positions of celestial bodies. Spherical triangles are solved by special rules of spherical trigonometry and contain important differences from planar triangles. In arc length operations on the Earth's surface, spherical triangle solutions are used, while in route determination operations, astronomical basic triangle solutions are used. With the astronomical basic triangle, the arc length between two points is determined by geographical coordinates (Şişman, 2024). Geographic coordinate information used in calculating the route distance of airports is given in Table 1.

Table 1. Location and identification information of airports

ICAO	IATA	Şehir	Havalimanı	Enlem (°)	Boylam (°)
LTFH	SZF	Çarşamba	Çarşamba	41°15'56"	36°32'55"
LBTJ	ADB	İzmir	Adnan Menderes	38°17'21"	27°09'18"
LTCF	KSY	Kars	Harakani	40°33'44"	43°06'53"
LTDB	COV	Adana	Çukurova	36°53'49"	35°04'03"

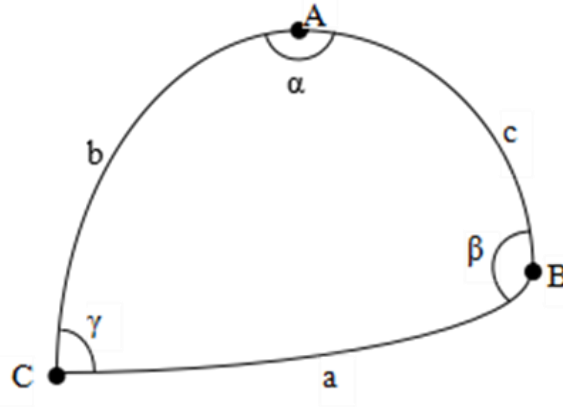


Figure 4. Representation of spherical triangle (Şişman, 2024)

In this study, the edge cosine theorem is applied for the calculation of the distance between the flight routes made between any two determined airports. Considering the point A shown in the figure as Samsun Çarşamba Airport and the point B as Kars Harakani Airport from the other selected airports, the c length value between them is obtained with the spherical coordinate information we have. The edge cosine theorem relates the length of a side to the other two sides and the angle between them. This relation is used to find any side of a spherical triangle. This theorem is used especially in spherical astronomy in calculating the distance between two points (Şişman, 2024).

$$\begin{aligned}\cos(a) &= \cos(b)\cos(c) + \sin(b)\sin(c)\cos(\alpha) \\ \cos(b) &= \cos(a)\cos(c) + \sin(a)\sin(c)\cos(\beta) \\ \cos(c) &= \cos(a)\cos(b) + \sin(a)\sin(b)\cos(\gamma)\end{aligned}$$

2.1.1.4. Spherical Radiates Used Instead of The Earth

Although the shape of the earth is generally accepted as an ellipsoid, in some calculations and small- scale map projects, the sphere surface is used without the need for an ellipsoid model. The most important element in spherical solution methods is the determination of a correct sphere radius (R). This radius can be defined by various formulas and is expressed in different ways below:

a) Gaussian Sphere Radius (R_G)

The radius of curvature R of the azimuthal normal section can be used to define the mean local radius of the ellipsoid. Although this is rarely applied, it is convenient when it is desired to locally approximate the ellipsoid by a sphere. This radius is the radius of the sphere being locally approximated.

For example, one type of mean local radius is the Gaussian mean radius, which is the average of the radii of curvature of all normal sections at a point.

The Gaussian mean radius is a function of latitude.

$$R_G = \frac{1}{2\pi} \int_0^{2\pi} R \, d\theta = \frac{1}{2\pi} \int_0^{2\pi} \frac{a^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta} \, d\theta$$

As a result of this integral, R_G is

$$R_G = \sqrt{\frac{a^2 b^2}{a^2 \sin^2 \phi + b^2 \cos^2 \phi}} = \frac{ab}{\sqrt{a^2 \sin^2 \phi + b^2 \cos^2 \phi}}$$

Is obtained as.

This formula mathematically expresses how the Radius



of a sphere relates to the ellipsoidal surface.

b) Radius of the Sphere Based on the Average of the Three Semi- Axes of the Ellipsoid (RM)

The radius of the sphere based on the average of the three semi- axes of the ellipsoid is the mean radius of curvature defined from the average of the principal curvatures.

$$R_M = \frac{1}{\frac{1}{2}(\frac{1}{N} + \frac{1}{M})}$$

Other mean radii are defined here that approximate the ellipsoid globally rather than locally. One is the mean of the semi- axes of the ellipsoid, and the other is the radius of the sphere whose surface area is equal to that of the ellipsoid.

In spherical analyses, under the rotational ellipsoid assumption, if the a and c axes are equal, the radius calculated for the sphere is obtained from the following formula:

$$R_M = \frac{a + b + c}{3} = \frac{1}{3} (2a + b)$$

c) Radius of the Sphere Covering the Same Area as the Ellipsoid (Ra)

The radius of a sphere having an area equal to the area of the ellipsoid can be calculated with the following formulas:

$$R_A = \sqrt{\frac{\Sigma}{4\pi}}$$

Here Σ is the area of the given ellipsoid. The area of the ellipsoid is calculated as follows:

$$\Sigma = 2\pi b^2 \left(\frac{1}{1-e^2} + \frac{1}{2e} \ln \frac{1-e}{1+e} \right)$$

d) Radius of Sphere with Same Volume as Ellipsoid (RV)

When the volume of the sphere V_s is equal to the volume of the ellipsoid V_e , the radius of the sphere is calculated as:

$$\begin{aligned} V_s &= \frac{4}{3}\pi R_v^3 \\ V_e &= \frac{4}{3}\pi a^2 b \\ R_v &= \sqrt[3]{a^2 b} \end{aligned}$$

e) Radius of the Sphere for Which the Sum of the Squares of the Distances from the Ellipsoid is Minimum (RJ)

The mean calculated by the surface integral of the geocentric radii defined by the geocentric latitude on the ellipsoid arrives at the following formula:

$$R_J = a \left(1 - \frac{1}{6}e^2 - \frac{11}{120}e^4 - \frac{103}{1680}e^6 + \dots \right)$$

f) Total Sphere Radius (Ro)

The sphere that covers an area equal to the ellipsoid surface area is called the "atal sphere". According to this definition, a atal sphere can be defined for each ellipsoid. In addition, the atal sphere can be considered as an average sphere that can replace a certain ellipsoid (Kimerling vd., 2016). The radius of the atal sphere is calculated as follows:

$$R_O = a \sqrt{\frac{q_N}{2}}$$

Here, q_N is the isometric latitude at the pole and this value is calculated when $\phi=90^\circ$. On the average sphere, the latitude is determined by the formula θ . On the ellipsoid, the isometric latitude is obtained from the formula q . The latitude value is calculated as follows:

$$\theta = \sin^{-1} \left(\frac{q}{q_N} \right)$$

Isometric latitude is found with the following formulas:

$$q = (1 - e^2) \left(\frac{\sin \varphi}{(1 - e^2 \sin^2 \varphi)} - \left(\frac{1}{2e} \right) \ln \left(\frac{1 - e \sin \varphi}{1 + e \sin \varphi} \right) \right)$$

Once the sphere radii are determined, two different methods can be used to calculate distance. The first method is to define points on the sphere surface using the geocentric coordinate system (Bildirici, 2016).

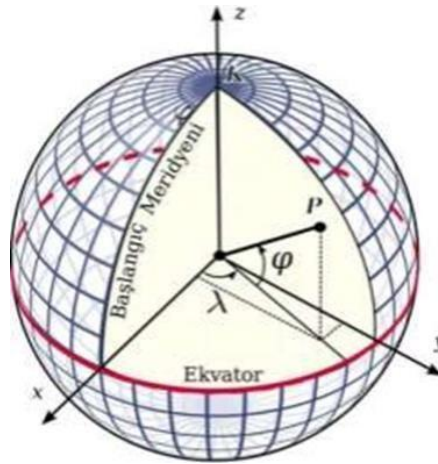


Figure 5. Sphere geocentric coordinate system

In this system, the y- axis is placed perpendicular to the other axes, and the z- axis is placed to work with the earth's rotation axis. The z- axis is chosen to pass through the point where the sphere center and the prime meridian intersect the equator (Bildirici, 2016). In this coordinate system, the relationship between the perpendicular coordinates of the points and the geographical coordinates is determined by the following formulas:

$$\begin{aligned} x &= (R + h) \cos \varphi \cos \lambda \\ y &= (R + h) \cos \varphi \sin \lambda \\ z &= (R + h) \sin \varphi \end{aligned}$$

These formulas are defined by the parametric equations of the sphere. If a certain point is not on the surface of the sphere and is located at a position below or above the surface, then the height along the radius of the sphere passing through that point should also be taken into account. The height value mentioned here can be selected as orthometric or ellipsoidal height (Bildirici, 2016).

Three - dimensional path length from geocentric coordinates is calculated as follows:

$$\sigma = \sum_{i=1}^{n-1} \sqrt{\Delta X_{i,i+1}^2 + \Delta Y_{i,i+1}^2 + \Delta Z_{i,i+1}^2}$$

In this formula, n represents the number of route points, i represents any route point, and Δ represents the coordinate difference (Bossler ve Snay, 2010).

In the second method, using spherical geographical coordinates, the shortest path length on the great circle arc (orthodromic curve) is calculated according to the following formula using the side cosine relation of the spherical triangle (Synder, 1987; Koçak, 1999; Uçar vd., 2011):

$$\sigma_{12} = R \cos^{-1}(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos(\lambda_2 - \lambda_1))$$

In this formula, comparisons can be made using the different sphere radii (Rg, Rm, Ra, Rv, Rj, Ro) mentioned above and the shortest route length on the sphere can be determined. When the average sphere radius is used, the shortest route length is calculated by the following formula, taking into account the isometric latitude:

$$\sigma_{12} = R_0 \cos^{-1}(\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 \cos(\lambda_2 - \lambda_1))$$



2.2. Material

There are 59 civil airports in operation in Türkiye.. While 37 of them carry out international flights, 18 of them are also used for military purposes. In the study, 8 airports located within the Turkish airspace were selected. Starting (departure) point was Samsun Çarşamba Airport, and ending (arrival) points were selected from one airport from each region of the country, and geodetic thermal problem solutions were performed. In this study, by comparing geodetic thermal problem solution methods on the globe, the airports we considered were taken as İzmir Adnan Menderes, Kars Harakani and Adana Çukurova Airports, and the aim was to determine the most appropriate solution methods for flight routes.

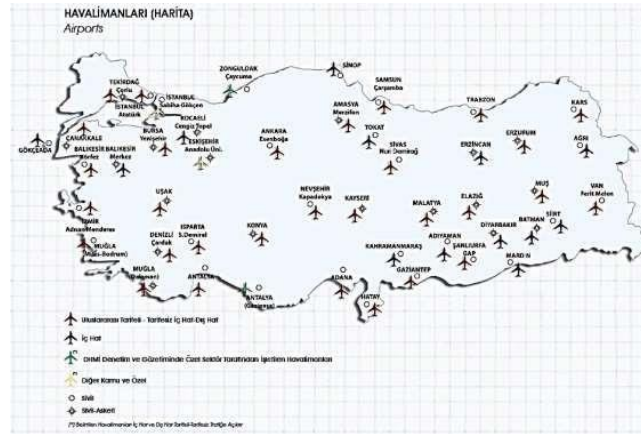


Figure 6. Airports in Turkey (DHMİ, 2017)

Samsun Çarşamba Airport, which we used as a starting point, was put into service in 1998. The airport's domestic terminal has an annual passenger capacity of 1.5 million and the international terminal has an annual passenger capacity of 500 thousand (URL_1). İzmir Adnan Menderes Airport, which we use as one of the arrival airports, was put into service in 1987 (URL_2). Kars Harakani Airport was put into service in 1988 (URL_4). Finally, Adana Çukurova International Airport became operational in 2024 after the end of Şakir Paşa Airport (URL_3). In addition, the 4 airports we examined have initiated the "Carbon- Free Airport Project" in order to control their possible impacts on the environment, to leave a more livable world for future generations with the measures taken against global warming and climate change, and to ensure sustainable airport operations.

Google Earth is a computer software developed by Keyhole, a company acquired by Google Labs, that displays photos of different resolutions taken from satellites around the world. While in July 2005, only relatively high-resolution photos of almost the entire United States were available, as of June 2006, detailed images of most cities in the world have been available.

3. Application and Results

In the study, firstly different sphere radii were calculated for the selected routes. Then, using these sphere radii, route lengths were determined with spherical trigonometry.

The route calculation was made using the different sphere radii (Table 2) that we examined

Table 2. Radius values of different spheres

Spheres	Sphere Radii (km)
R_g	6.375,318
R_m	6.378,161
R_a	6.371.007
R_v	6.371,001
R_j	6.370,994
R_o	6.371,007



R_g, R_m, R_a, R_v, R_j is intended to obtain route lengths using different sphere R radii and R_o is intended to obtain route lengths using authentic sphere radius. Samsun Çarşamba Airport was taken as the starting airport from the airports used in the study area, İzmir Adnan Menderes Airport, Adana Çukurova International Airport and Kars Harakani Airport were accepted as the destinations and geodetic curve images of flight routes were obtained in KML format via Google Earth Pro.

The distance calculation between two airports was made on the sphere with the coordinate information of the airports we have and the sphere radii and the results were compared. Thus, it was aimed to determine the most appropriate solution method for flight routes. The route lengths obtained using different sphere radii are shown in Table 3.

Table 3. Distance values of flight routes on the sphere

Flight Route	Samsun –İzmir (km)	Samsun –Kars (km)	Samsun-Adana (km)
Sphere			
R_g	868,291	557,500	502,625
R_m	868,678	557,748	502,849
R_a	867,704	557,123	502,285
R_v	867,703	557,122	502,285
R_j	867,702	557,122	502,285
R_o	868,960	558,164	501,908

Table 3 shows the flight path lengths obtained using different sphere R radii (R_g, R_m, R_a, R_v, R_j), while R_o shows the path length found using the authentic sphere radius.

Using spherical geographic coordinates, the shortest path length on the great circle arc (orthodromic curve) was calculated using the side cosine relation of the spherical triangle according to the following formula:

$$\sigma_{12} = R \cos^{-1}(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos(\lambda_2 - \lambda_1))$$

In this formula, comparisons can be made using the different sphere radii (R_g, R_m, R_a, R_v, R_j, R_o) mentioned above and the shortest route length on the sphere can be determined.

When the otalic sphere radius is used, the shortest route length is calculated by taking the isometric latitude into account with the following formula:

$$\sigma_{12} = R_o \cos^{-1}(\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 \cos(\lambda_2 - \lambda_1))$$

Comparison of route lengths obtained using different radii (R_g, R_m, R_a, R_v, R_j, R_o) was made in Table 3 and it was seen that the results were similar and the method that gave the best results in terms of distance was determined.

After calculating the route distances, KML format images were created with the Google Earth Pro program.

Samsun Çarşamba Airport was taken as the starting airport among the airports used in the study area and İzmir Adnan Menderes Airport (Figure 7), Kars Harakani Airport (Figure 8) and Adana Çukurova International Airport (Figure 9) were taken as the arrival airports.



Geodetic curve representation of the airports examined was presented in KML format.

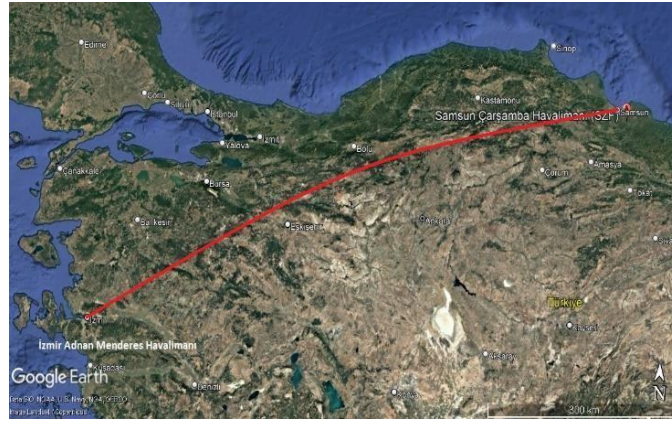


Figure 7. Geodetic curve image between Samsun-Izmir

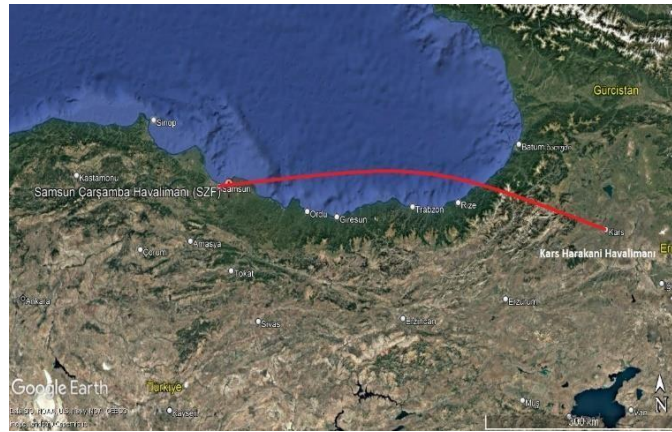


Figure 8. Geodetic curve image between Samsun-Kars

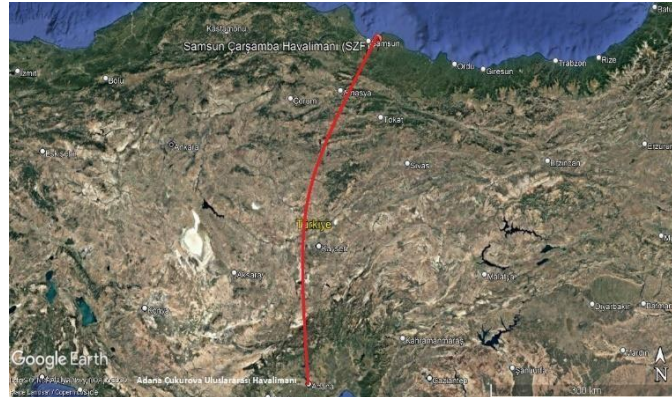


Figure 9. Geodetic curve image between Samsun-Adana

4. DISCUSSION

The findings of the study are that the global air transportation system is a pioneer in the world economy and is constantly growing. It is predicted that the air traffic increase predicted by airline companies between 2016-2035 will be between 3-6% with regional differences (Çelik, K.; Eren, H., 2020). However, in order to continue the continuous growth, the Air Traffic Management (ATM) systems used today need to be updated and largely transformed. The basis of this transformation is the transition from the currently used clearing-based Air Traffic Control (ATC) operations to orbit-based operations. Since increasing air traffic makes the job of pilots and controllers difficult, it is necessary to research programs that need to be developed such as NextGen and SESAR, which make orbital predictions. Such programs should not be limited to the US and the EU, and the work of more countries in the field of global air transportation should be examined and it should be aimed to direct orbit-based operations.



5. CONCLUSIONS

Within the scope of this study, the concept of 4D orbit was first discussed. Today, the density of air traffic and the growth of the aviation sector increase the difficulty of air traffic and prevent it from working efficiently. It is predicted that the 4D orbit application will allow the accurate prediction of the impact it will create in the near future by reducing the workload of pilots and controllers by researching NextGen and SESAR. The route is created by determining 3 different flight routes with the coordinate information of 4 different airports and calculating the distance between the airports on both the sphere and the ellipsoid. The average, and standard deviation values of the Samsun - Izmir, Samsun - Kars and Samsun - Adana routes were calculated. (Table 4). The results shown in the table were obtained by calculating the 3 route distance lengths with 6 different sphere radii

Table 4: Average and Standard Deviation values of Flight Routes

	Average (km)	Standard Dev.(km)
Samsun- İzmir	868,173	0,557
Samsun- Kars	557,463	0,429
Samsun- Adana	502,373	0,325

Also, in the next studies, the values of the created flight route lengths can be compared as the shortest and most suitable distance for the different spheres and ellipsoids. Thus, fuel-saving calculations can be made from the distance difference between the different spheres, and which surface flight was more suitable for fuel consumption can be obtained. In addition, the distance calculated on the sphere and the ellipsoid and the fuel consumption rate of the aircraft per km can be determined.

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Author contributions

The contributions of the authors to the article should be stated.

Conflicts of interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

Research and publication ethics were complied with in the study.

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URL_Figure_1 [4D Trajectory add the 4th dimension of time to flight plans].

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