

Research Article /Review Article:

## Fibonacci-Anchored Approach to Road Curve Design: Golden Curvature with Single-Segment and $G^2$ -Continuous Composite Construction Modes

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### Abstract

Horizontal road curve design has traditionally relied on prescribed curvature laws (circular arcs, clothoids, and clothoid-arc-clothoid (SCS) hybrids) to satisfy geometric feasibility and driving-comfort requirements. These classical approaches are based on fixed or linearly evolving curvature profiles and provide limited control over how curvature is redistributed along a curve under given design constraints. This study proposes a Fibonacci-anchored curvature formulation, referred to as Golden Curvature (GC), as a parametric and curvature-continuous alternative for horizontal alignment design. The framework is governed by a single dimensionless intensity parameter  $\beta$  and is realized in two distinct construction modes: a Single-Segment GC in which curvature varies monotonically between two non-zero values, and a Composite Golden Curvature Spiral (GCS) in which two single-GC half-segments are joined symmetrically (optionally through a central circular arc) so that  $\kappa(0) = \kappa(L) = 0$ , delivering  $G^2$ -continuous tangent-to-tangent transitions. Asymptotic analysis demonstrates that the framework recovers the circular arc ( $\beta \rightarrow 0$ ), the clothoid ( $\beta = \log F_N$ ), and the logarithmic spiral ( $\beta = 1, N \rightarrow \infty$ ) as analytically identifiable limit cases within a single parametric family. Three case studies, moderate horizontal curves at  $\Delta\theta = 68^\circ$  and  $90^\circ$ , and a cloverleaf interchange at  $\Delta\theta = 270^\circ$  per loop, are evaluated under AASHTO Green Book [1] design-speed scenarios. The Single-Segment GC achieves 25–45% shorter curve length and up to 65% smaller spatial footprint than the SCS hybrid at moderate deflections; at high deflections ( $270^\circ$  cloverleaf loops), the small- $\beta$  regime of the Single-Segment GC approaches the pure-circular geometric optimum to within 2% while satisfying the AASHTO and Lamm-et-al. operational comfort criteria. Vehicle-dynamics, side-friction, and constructability analyses confirm that the framework integrates cleanly into established design practice. The proposed Golden Curvature framework is positioned as a continuous parametric extension of the classical clothoid-based toolkit, rather than a replacement of it.

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### Contribution to the field statement:

This study introduces a Fibonacci-anchored Golden Curvature framework for horizontal road design, offering two construction modes that balance geometric compactness and curvature continuity. By recovering circular, clothoidal, and logarithmic spiral behaviours as limiting cases, it expands the classical curve-design toolkit and demonstrates potential space savings through numerical case studies.

### Highlights:

- A Fibonacci-based framework introduces a new approach to road curve design.
- Two construction modes address compactness and curvature continuity.
- The method recovers circular, clothoidal, and logarithmic spiral behaviours.
- Single-segment curves reduce length by up to 45% in tested cases.

### Cite this article?

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## 1. Introduction

The geometric design of road and vehicle trajectories is a fundamental aspect of transportation engineering, autonomous-driving path planning, and infrastructure safety. In practice, horizontal alignments are formed by combining straight segments, circular arcs of constant curvature, and transition curves that provide a gradual change in curvature between these elements. Among the available transition-curve formulations, the clothoid (also known as Euler spiral or Cornu spiral) has become the dominant standard in highway and railway design, because its curvature changes linearly with arc length and therefore yields the smoothest possible build-up of lateral acceleration under the variational jerk-energy criterion [2–4].

Despite its long-standing acceptance, the clothoid presents several well-documented limitations. Mathematically, it is defined through Fresnel integrals which do not admit closed-form expressions and require numerical evaluation or approximation in practical implementation [5, 6]. For these reasons, polynomial and rational-Bézier approximations of clothoids have been extensively studied [3, 4, 7], and alternative families of transition curves with different curvature-evolution rules have been proposed [8–13]. However, most such alternatives remain mathematically close to the clothoid and differ from it primarily in approximation strategy rather than in the underlying curvature-growth law.

In a separate research thread, curve families derived from Fibonacci sequences and golden-ratio scaling have been studied in contexts of self-similar geometry, logarithmic-spiral construction, optimal packing, and growth-driven form generation [14, 15]. Fibonacci-based formulations naturally yield non-linear, scale-driven progression and are well suited to controlled redistribution of geometric properties. Despite this, their potential role as a parametric curvature-control mechanism in road and trajectory design has not been systematically explored.



Figure 1. Examples of spiral geometry as Fibonacci Curves across nature and science. Atmospheric cyclones, spiral galaxies, biological shells, mathematical spiral constructions, and plant phyllotaxis

This study addresses that gap by proposing a Fibonacci-anchored curvature formulation, termed Golden Curvature (GC), as a parametric and curvature-continuous alternative for horizontal alignment design. Unlike classical clothoids, which enforce a strictly linear curvature-arc-length relationship, Golden Curvature encodes a smooth multiplicative curvature evolution governed by a single dimensionless parameter  $\beta$ , and embeds the circular arc ( $\beta \rightarrow 0$ ), the clothoid ( $\beta = \log F_N$ ), and the logarithmic spiral ( $\beta = 1, N \rightarrow \infty$ ) as analytically identifiable limit cases. Two construction modes are derived from the same underlying curvature law: a Single-Segment GC for compactness-oriented applications with  $G^1$  tangent-curve continuity, and a Composite Golden Curvature Spiral (GCS) (directly analogous in topology to the SCS hybrid) for  $G^2$ -continuous tangent-to-tangent transitions.

Three case studies are analyzed under the AASHTO Green Book [1] design-speed framework: two moderate horizontal curves at  $\Delta\theta = 68^\circ$  and  $\Delta\theta = 90^\circ$ , and a four-leaf cloverleaf interchange at  $\Delta\theta = 270^\circ$  per loop. Vehicle-dynamics, side-friction, and constructability analyses complement geometric comparison. The results demonstrate that the Golden Curvature framework provides controllable additional flexibility in the compactness-comfort trade-off, with measurable benefits in geometric compactness at moderate deflections and in approaching the pure-circular geometric optimum at high deflections while maintaining monotone curvature build-up. The framework is positioned as a supplementary, standards-compatible tool within the existing horizontal-alignment design workflow rather than a replacement for it.

The remainder of the paper is organized as follows. Section 2 reviews classical transition-curve formulations and related work. Section 3 develops the mathematical foundations of the proposed framework and formally defines the two construction modes. Section 4 specifies the step-by-step implementation. Section 5 reports empirical evaluation and engineering analyses for the three case studies. Section 6 discusses positioning, trade-offs, limitations, and future work. Section 7 concludes.



## 2. Related Work

### 2.1. Classical transition curves and the dominance of the clothoid

Classical road curve design is governed by predefined curvature prescriptions intended to ensure vehicle stability, driving comfort, and operational safety [16, 17]. The circular curve is the most basic and widely applied curve model, defined by constant curvature  $\kappa = 1/R$  along the entire length; it offers computational simplicity but introduces curvature discontinuities at the tangent–curve interface, motivating the use of transition curves for comfort and safety.

Among the available transition-curve formulations, the clothoid has emerged as the dominant solution in highway and railway engineering owing to its linear curvature–arc-length relationship [3]. This property directly limits lateral acceleration and jerk and follows from the variational optimality of the clothoid for the jerk-energy functional under fixed boundary curvatures and prescribed deflection. The mathematical formulation relies on Fresnel integrals [4, 5], which do not admit closed-form expressions and require numerical or approximate evaluation in practice. McCrae and Singh [2] emphasise the importance of curvature-continuous ( $G^2$ ) transitions in interactive curve sketching but note that strict adherence to clothoidal geometry can restrict design flexibility in complex or space-constrained layouts [2, 18].

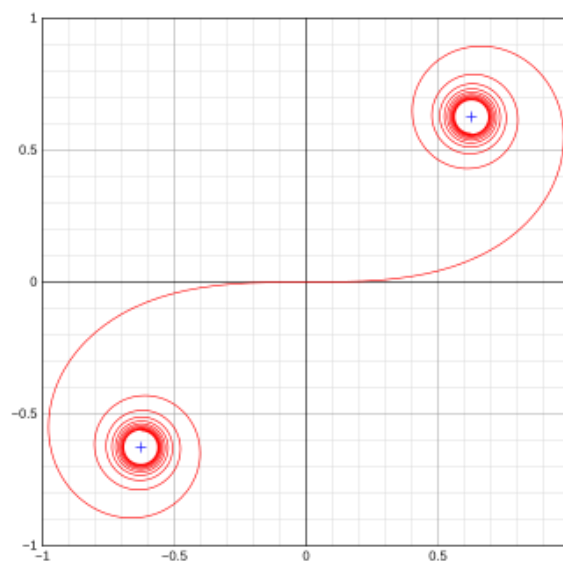


Figure 2. Clothoid Spiral

### 2.2. Computational alternatives and approximation strategies

Because Fresnel integrals are analytically complex, much of the subsequent literature has focused on approximating clothoids with polynomial or parametric curves [3, 6]. Free-form-curve approximations preserve the principal curvature behaviour of the clothoid while enabling efficient computation [4]. Spline-based methods provide computationally convenient alternatives but cannot fully reproduce linear curvature change [10]. Reif and Weinmann [19] examine geometric Hermite subdivision based on clothoidal primitives, while Chen et al. [7] report accurate clothoid approximation using rational Bézier curves for path planning. A recurring observation in this literature is that the clothoid is treated as an ideal geometric target rather than as part of a broader, more flexible design framework and that exploration of fundamentally different curvature-evolution laws has remained limited.

### 2.3. Alternative transition-curve formulations

As the limitations of classical clothoids have become more apparent, several studies have introduced alternative transition-curve families with different curvature profiles. Eliou and Kaliabetsos [9] propose a transition curve based on symmetric projection that preserves continuity and smoothness while offering greater geometric flexibility. Pan et al. [12] introduce the K-curve, a generalisation of the clothoid controlled by the ratio between chord-tangent and tangential angles, yielding different lateral-force characteristics. Kobryń [11] applies polynomial transition curves to vertical alignment design, and Basak [8] and Zhang et al. [13] study related geometric formulations in the context of circular roads and intersections. These contributions demonstrate that the concept of a transition curve is not uniquely tied to the clothoid; instead it depends on the broader criteria of curvature continuity, bounded acceleration, and constructability. Nevertheless, most proposed alternatives remain mathematically close to the clothoid family and do not invoke a fundamentally different curvature-growth law.

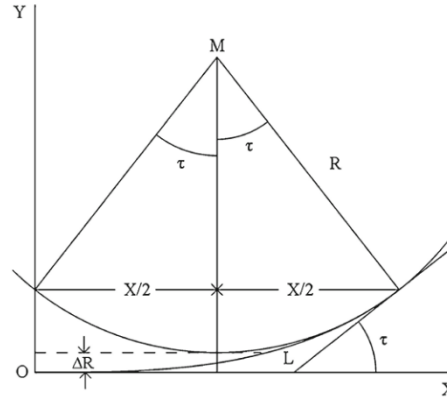


Figure 3. Symmetrically Projected Transition Curve (SPTC) [9]

## 2.4. Fibonacci-based geometry in engineering

Fibonacci sequences and golden-ratio scaling appear across mathematics, physics, and engineering in problems involving self-similar geometry, logarithmic spirals, optimal packing, and growth-driven form generation [14, 15]. Such formulations naturally provide smooth, non-uniform progression and have been employed for hierarchical load distribution, structural rib design, and natural-pattern synthesis. Their potential role as a parametric curvature-control mechanism in road and trajectory design, however, has not been systematically investigated. The present work addresses that gap and positions Fibonacci anchoring as a discrete control mechanism for an exponential curvature law that recovers classical curve families as limit cases.

## 2.5. Highway-engineering design standards

Practical horizontal-alignment design is governed by national and international standards. The AASHTO A Policy on Geometric Design of Highways and Streets [1] is the most widely adopted reference, providing minimum-radius tables, side-friction factors, and superelevation requirements indexed by design speed. European national standards (RAS-L Germany [20], DMRB United Kingdom [21], ICTAAL France [22]) follow comparable formulations rooted in the same equilibrium relation  $R_{min} = \frac{v^2}{127(e+f_{max})}$ . Lamm, Psarianos and Mailaender [23] provide a comprehensive treatment of safety and comfort criteria, including operational thresholds on lateral acceleration and jerk that guide the analyses of Section 5. The Golden Curvature framework proposed here operates entirely within these standard input parameters and is compatible with the standards-based design workflow.

## 3. Methodology

This section establishes the mathematical foundations of the proposed Fibonacci-anchored curvature framework, referred to as Golden Curvature (GC), and identifies two distinct construction modes derived from a common curvature law. The first mode, termed Single GC, realises the framework as a single monotone spiral and is the elementary formulation; the second, termed Composite GCS (Golden Curvature Spiral-composite), is a symmetric two-segment construction designed to deliver  $G^2$ -continuous tangent-to-tangent transitions. The two modes share a common asymptotic structure and are presented side by side throughout the manuscript.

The formulation is developed in a non-dimensional domain to isolate the intrinsic geometric behaviour of curvature evolution from physical scale effects. Let  $L$  denote the total curve length and introduce the normalised arc-length variable

$$\hat{s} = s / L, \quad \hat{s} \in [0, 1]. \quad (1)$$

This transformation maps all curves onto a common unit interval, enabling direct and fair comparison between different curvature laws independently of physical length, design speed, or unit system.

### 3.1. Mathematical foundations of Fibonacci numbers

The Fibonacci sequence  $F_n$  is defined recursively as  $F_0 = 0$ ,  $F_1 = 1$ ,  $F_{n+2} = F_{n+1} + F_n$  ( $n \geq 0$ ). Binet's formula gives the closed-form expression  $F_n = \frac{\varphi^n - \psi^n}{\sqrt{5}}$ ,  $\varphi = \frac{1+\sqrt{5}}{2}$ ,  $\psi = \frac{1-\sqrt{5}}{2}$ , where  $\varphi = (1+\sqrt{5})/2$  and  $\psi = (1-\sqrt{5})/2$ . The sequence grows exponentially as  $F_n = \theta(\varphi^n)$ , and the ratio of consecutive terms converges to the golden ratio:

$$\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = \varphi. \quad (2)$$

There exists  $C > 0$  such that  $|F_{n+1}/F_n - \varphi| \leq C \cdot |\psi/\varphi|^n$ , implying exponential convergence of Fibonacci ratios toward the golden ratio. This property is the structural reason why a curvature law built on Fibonacci anchoring inherits asymptotically golden-ratio-consistent scaling.



### 3.2. Curvature-based curve representation

Let  $\gamma : [0, L] \rightarrow \mathbb{R}^2$  be a planar curve parameterised by arc length  $s$ , with unit tangent  $T(s) = \gamma'(s)$ . The curvature  $\kappa(s)$  is defined through the Frenet relation  $T'(s) = \kappa(s) N(s)$ ; introducing the tangent angle  $\theta(s)$ , one has  $\gamma'(s) = (\cos \theta(s), \sin \theta(s))$  and  $\theta'(s) = \kappa(s)$ .

For  $\kappa \in L^1([0, L])$ , the curve  $\gamma$  exists, is absolutely continuous, and satisfies  $\|\gamma'\| = 1$  almost everywhere, with

$$\theta(s) = \theta_0 + \int_0^s \kappa(u) du, \quad \gamma(s) = \gamma(0) + \int_0^s (\cos \theta, \sin \theta) du. \quad (3)$$

If  $\kappa$  is continuous,  $\gamma \in C^2([0, L])$  with curvature  $\kappa(s)$ . This result establishes that specifying a curvature function is sufficient to uniquely define a planar curve up to rigid motion.

### 3.3. Fibonacci-anchored scaling and asymptotic structure

Within the non-dimensional formulation, define a strictly monotone scaling map  $\hat{\theta} : [0, 1] \rightarrow [0, 1]$  that interpolates the discrete pairs  $(\hat{s}_n, \log F_n / \log F_N)$ , where the Fibonacci-anchored nodes are

$$\hat{s}_n = \frac{\sum_{k=1}^n F_k}{\sum_{k=1}^N F_k}, \quad n = 1, \dots, N. \quad (4)$$

A monotone cubic Hermite interpolant (PCHIP) is used to guarantee  $\hat{\theta}'(\hat{s}) \geq 0$  and to prevent overshoot. The non-dimensional Golden Curvature is then defined as

$$\hat{\kappa}(\hat{s}) = \exp(\beta \hat{\theta}(\hat{s})), \quad \beta > 0. \quad (5)$$

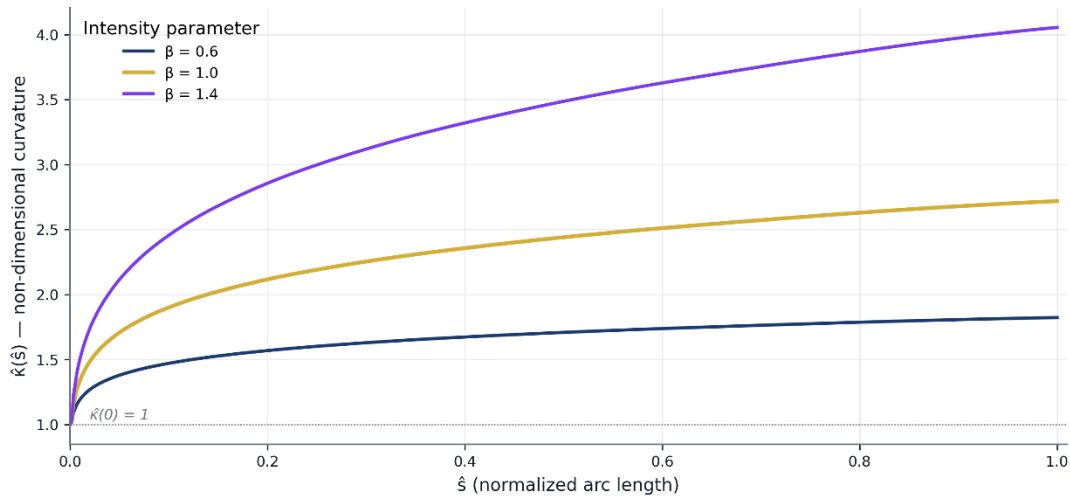


Figure 4. Non-dimensional Golden Curvature  $\hat{\kappa}(\hat{s})$  for three intensity values  $\beta$ . Larger  $\beta$  concentrates curvature toward the apex of the unit interval.

By construction, the discrete-node ratios satisfy

$$\frac{\hat{\kappa}(\hat{s}_{n+1})}{\hat{\kappa}(\hat{s}_n)} = \left( \frac{F_{n+1}}{F_n} \right)^\beta \rightarrow \varphi^\beta \quad \text{as } n \rightarrow \infty. \quad (6)$$

This is the formal sense in which the proposed curve family is Fibonacci-consistent: at the discrete anchor nodes the curvature ratio converges to the golden ratio raised to the intensity exponent.

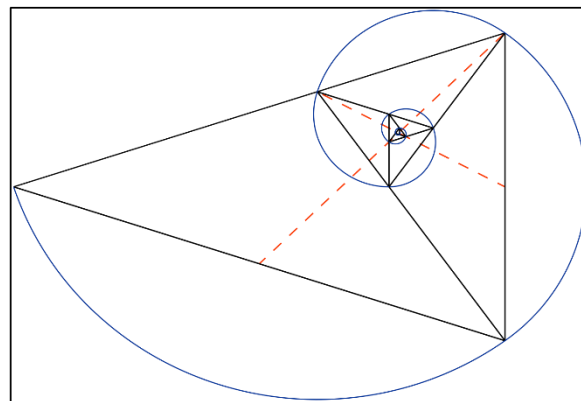


Figure 5. Illustration of logarithmic spiral geometry associated with Fibonacci scaling

An asymptotic analysis clarifies the global behaviour of the resulting curve. For large  $N$ , the sum identity  $\sum_{k=1}^n F_k = F_{n+2} - 1$  and the relation  $F_n \sim \varphi^n / \sqrt{5}$  yield  $\hat{s}_n \approx \varphi^{(n-N)}$  and  $\log F_n \approx n \log \varphi$ . Inverting and substituting into (5) gives the asymptotic identity



$$\hat{\kappa}(\hat{s}) \approx \exp(\beta) \cdot \hat{s}^\alpha, \quad \alpha = \frac{\beta}{\log F_N}, \quad \text{on } (0,1]. \quad (7)$$

Three special parameter regimes are immediately apparent:

(i)  $\beta \rightarrow 0 \Rightarrow \alpha \rightarrow 0 \Rightarrow \hat{\kappa} \rightarrow \text{constant}$ . After dimensional rescaling with  $\kappa_{\max}$ , this is the circular arc.

Limit-case I: GC  $\rightarrow$  Circular Arc ( $\Delta\theta=68^\circ$ )

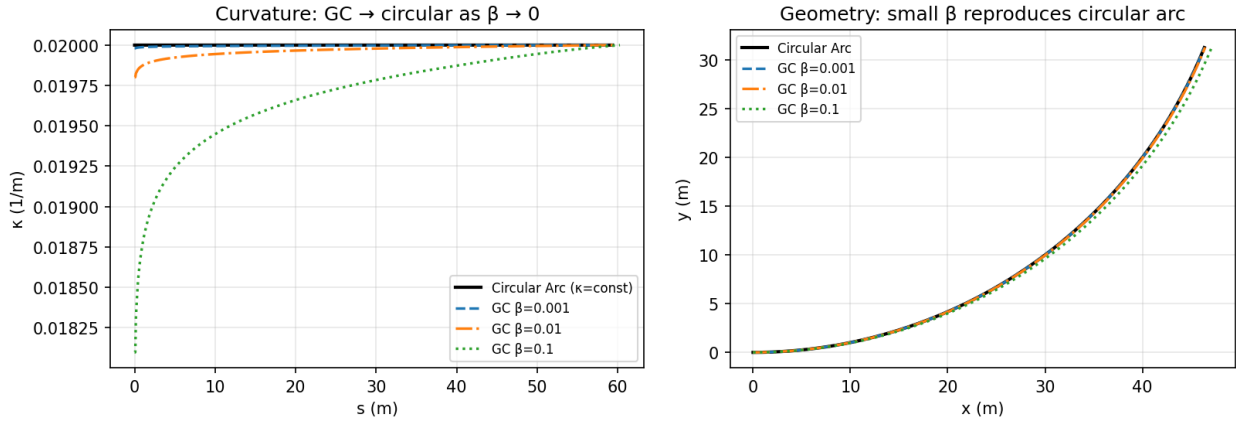


Figure 6. Circular limit case: as  $\beta \rightarrow 0$  the Golden Curvature profile and trajectory converge to the constant-curvature circular arc

(ii)  $\beta = \log F_N \Rightarrow \alpha = 1 \Rightarrow \hat{\kappa} \propto \hat{s}$ . Up to scaling, this is the defining property of the clothoid; the Fibonacci-anchored curve reproduces classical linear curvature growth at this specific  $\beta$ .

Limit-case II: GC  $\rightarrow$  Clothoid ( $\Delta\theta=68^\circ$ )

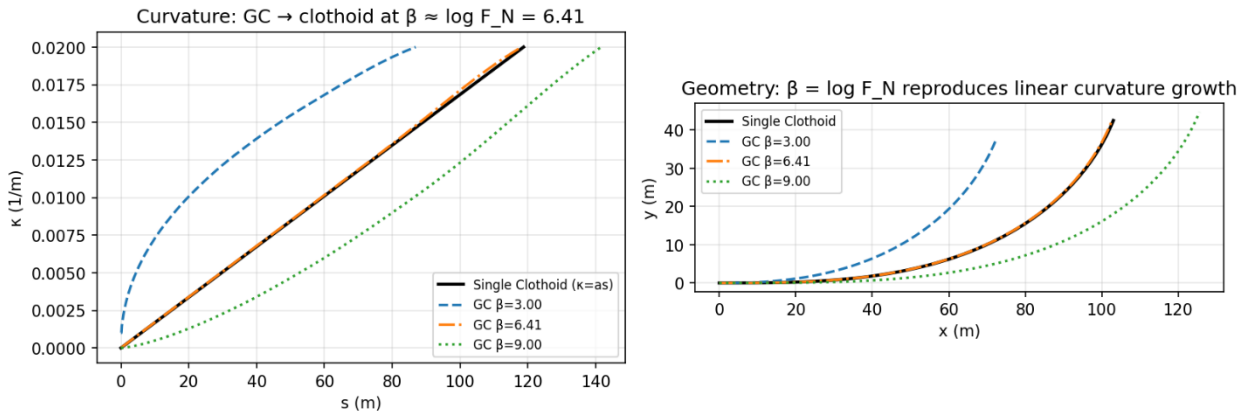


Figure 7. Clothoid limit case: at  $\beta = \log F_N \approx 6.41$  ( $N = 15$ ), the Golden Curvature profile reproduces the linear curvature growth of the single clothoid to within 1% on every metric

(iii)  $\beta = 1$  with  $N \rightarrow \infty$  recovers the logarithmic spiral: ratios at successive Fibonacci nodes converge exactly to  $\phi$ , the defining property of a logarithmic spiral with growth rate  $\log \phi$ .

These three classical curves, circular arc, clothoid, and logarithmic spiral, are recovered as analytically identifiable special cases of the same family. For finite  $N$  and  $\beta$  values outside these limits, GC interpolates continuously between them, providing a unified parametric framework rather than an alternative to the classical curves.

### 3.4. Definition of Golden Curvature: two construction modes

The exponential curvature law (5) admits two distinct geometric realisations, each suitable for a different class of engineering applications. The following definition formalises the framework and explicitly introduces both modes.

**Definition 1 (Golden Curvature framework).** A planar curve is said to follow the Golden Curvature (GC) framework if its non-dimensional curvature is generated by the law  $\hat{\kappa}(\hat{s}) = \exp(\beta \hat{\theta}(\hat{s}))$ , with  $\beta > 0$  and  $\hat{\theta}: [0,1] \rightarrow [0,1]$  a monotone scaling map constructed so that its samples at Fibonacci-anchored nodes satisfy (6). Two geometric construction modes are derived from this law:

**Mode A — Single GC.** The curvature law (5) is applied directly on a single arc-length interval  $[0, L]$ . After dimensional scaling by  $\alpha = \kappa_{\max}/\max(\hat{\kappa})$ , the resulting curve has curvature varying monotonically from  $\kappa_{\min} = \kappa_{\max} \cdot \exp(-\beta)$  at  $\hat{s} = 0$  to  $\kappa_{\max}$  at  $\hat{s} = 1$ . Single GC is the elementary GC building block and is appropriate when  $G^1$  continuity is sufficient or when the GC segment is embedded inside a larger construction.

**Mode B — Composite GCS.** Two Single-GC half-segments are joined symmetrically: optionally through a central circular core of radius  $R_{\min} = 1/\kappa_{\max}$  — so that  $\kappa(0) = \kappa(L) = 0$ . The composite consists of three segments: an entry half-spiral, an optional constant-curvature arc, and an exit half-spiral mirroring the entry. The Composite



GCS is the direct GC analogue of the clothoid-based SCS hybrid and is appropriate when curvature continuity at the tangent–curve interface is the design priority.

Both modes share the underlying curvature law (5), the same Fibonacci anchoring, the same intensity parameter  $\beta$ , and the same asymptotic structure of (7). They differ only in how the law is realised across the design interval. The labels ‘Single GC’ and ‘Composite GCS’ are used consistently in the empirical comparisons of Section 5 to distinguish the two modes; the generic term ‘Golden Curvature’ (GC) is used when a statement applies to both.

#### 4. Implementation in Road Design

The previous section established the analytical foundation of Golden Curvature and defined two construction modes. This section translates the framework into an implementable curve-generation procedure suitable for road geometry. The steps are illustrated in Fig. 8 as an infographic. Steps 4.1–4.10 describe the implementation of the Single GC mode as a self-contained ten-step algorithm. Subsection 4.11 extends this procedure to the Composite GCS mode, giving the additional construction rules required for  $G^2$ -continuous tangent transitions.

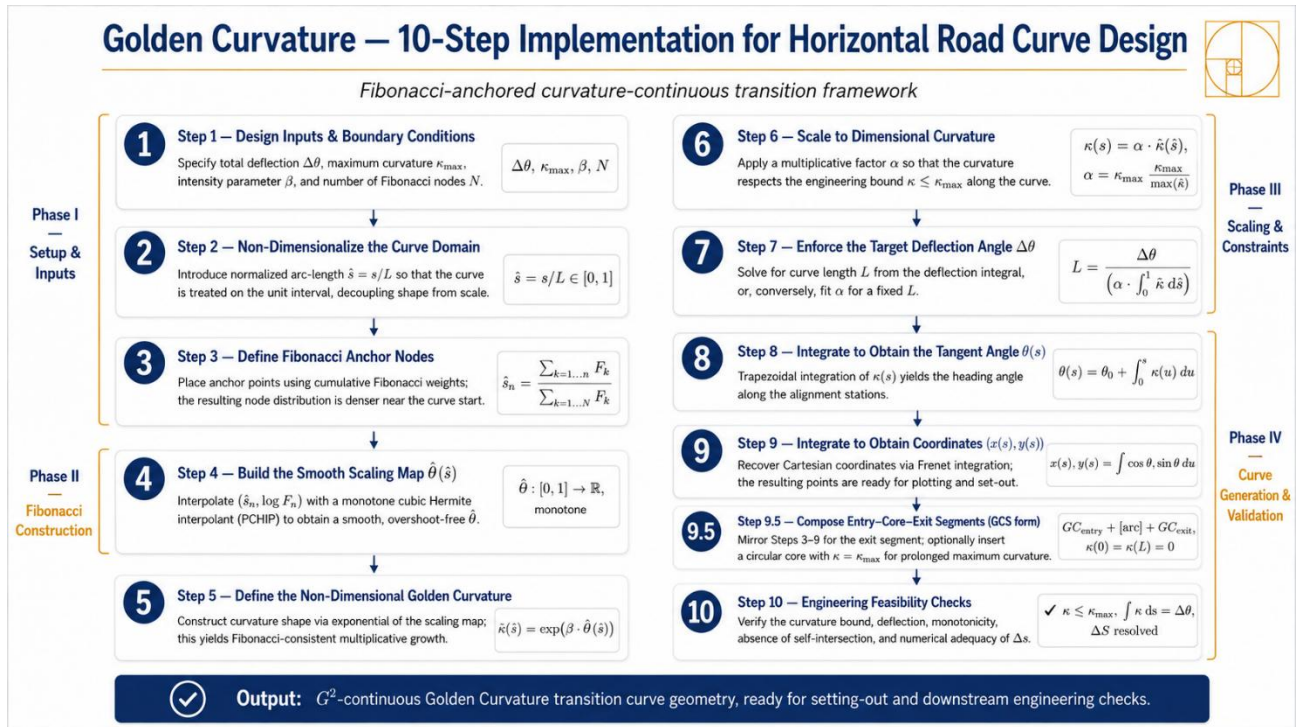


Figure 8. Implementation flowchart for the Single GC mode: ten-step procedure organised into four phases (setup, Fibonacci construction, scaling & constraints, curve generation & validation)

##### 4.1. Step 1 — Design inputs and boundary conditions

A Golden Curvature design is specified by four primary inputs: the total deflection angle  $\Delta\theta$ , the maximum allowable curvature  $\kappa_{\max}$ , the intensity parameter  $\beta$ , and the number of Fibonacci anchor nodes  $N$ . The deflection and the curvature bound are treated as hard constraints derived from the alignment problem;  $\beta$  and  $N$  are tuning parameters of the GC family. Under standard design practice [1], the curvature bound is obtained from  $\kappa_{\max} = 1/R_{\min}$  with  $R_{\min} = \frac{v^2}{127(e+f_{\max})}$ ; the values of  $v$ ,  $e$ , and  $f_{\max}$  are dictated by the design context.

##### 4.2. Step 2 — Non-dimensionalise the curve domain

Introduce  $\hat{s} = s/L \in [0, 1]$ , which decouples geometric shape from absolute scale and enables direct comparison across curve families under the same  $\Delta\theta$  and  $\kappa_{\max}$ .

##### 4.3. Step 3 — Define Fibonacci anchor nodes

With  $F_1 = F_2 = 1$ ,  $F_i = F_{i-1} + F_{i-2}$  define the Fibonacci-anchored node locations in  $[0, 1]$  via the normalised cumulative weights of (4). This construction places anchor nodes densely near  $\hat{s} = 0$ , providing strong anchoring of early curvature evolution and allowing controlled back-loading of curvature toward the end of the segment.

##### 4.4. Step 4 — Build the smooth monotone scaling map $\hat{\theta}(\hat{s})$

Construct  $\hat{\theta}: [0, 1] \rightarrow [0, 1]$  by monotone cubic Hermite interpolation (PCHIP) through the discrete pairs  $(\hat{s}_n, \log F_n / \log F_N)$ . PCHIP guarantees  $\hat{\theta}'(\hat{s}) \geq 0$  and prevents overshoot, which would otherwise introduce non-physical oscillations in the curvature profile. The logarithmic anchoring is what produces the Fibonacci-consistent multiplicative growth identified in (6).



#### 4.5. Step 5 — Define the non-dimensional Golden Curvature

Apply (5):  $\hat{\kappa}(\hat{s}) = \exp(\beta \hat{\theta}(\hat{s}))$ . Interpretation of  $\beta$ :  $\beta = 1$  corresponds to strict golden-ratio scaling at Fibonacci nodes;  $\beta < 1$  yields softened (compactness-oriented) curvature growth approaching a constant profile;  $\beta > 1$  yields stronger curvature concentration;  $\beta = \log F_N$  reproduces clothoid-like linear curvature.

#### 4.6. Step 6 — Scale to dimensional curvature

Apply a multiplicative factor  $\alpha$  to obtain dimensional values that respect the engineering bound:

$$\alpha = \frac{\kappa_{max}}{\max_{\hat{s} \in [0,1]} \hat{\kappa}(\hat{s})}, \quad \kappa(s) = \alpha \cdot \hat{\kappa}(\hat{s}). \quad (8)$$

By construction  $0 \leq \kappa(s) \leq \kappa_{max}$ .

#### 4.7. Step 7 — Enforce the target deflection angle

The total tangent-angle change is  $\Delta\theta = \alpha L \int_0^1 \hat{\kappa}(\hat{s}) d\hat{s}$ . Two design options are standard. Case 1 (length as free variable):

$$L = \frac{\Delta\theta}{\alpha \int_0^1 \hat{\kappa}(\hat{s}) d\hat{s}}. \quad (9)$$

Case 2 (length fixed): solve for  $\alpha$  at fixed  $L$  and verify the  $\kappa_{max}$  bound. In road-design workflows it is common to treat  $\kappa_{max}$  and  $\Delta\theta$  as hard constraints and to tune  $(L, \beta)$  accordingly.

#### 4.8. Step 8 — Integrate to obtain the tangent angle

With the curvature profile fully specified, the heading angle is obtained by quadrature  $\theta(s) = \theta_0 + \int_0^s \kappa(u) du$ , evaluated numerically by trapezoidal integration on a uniform station grid.

#### 4.9. Step 9 — Integrate to obtain coordinates

Planar coordinates follow from Frenet kinematics:

$$x(s) = x_0 + \int_0^s \cos\theta(u) du, \quad y(s) = y_0 + \int_0^s \sin\theta(u) du. \quad (10)$$

Trapezoidal integration over  $M$  stations yields the set-out points  $(x_j, y_j)$ , directly usable for alignment representation, GPS/GNSS staking, and downstream engineering analysis.

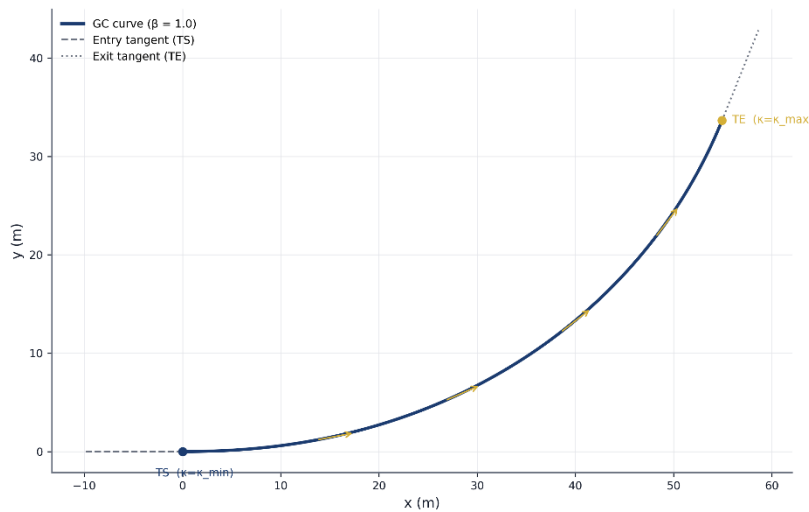


Figure 9. Single GC final geometry for  $\Delta\theta = 68^\circ$ ,  $\beta = 1.0$ : heading vectors along the alignment, entry tangent TS and exit tangent TE marked

#### 4.10. Step 10 — Engineering feasibility checks

Verify three criteria: (i) constraint compliance, with  $\max \kappa(s) \leq \kappa_{max}$  and  $\int_0^L \kappa(s) ds = \Delta\theta$  within numerical tolerance; (ii) geometric validity, including monotonicity of  $\kappa(s)$  where required and absence of self-intersection or unintended inflection within the deflection range; (iii) numerical adequacy, with station spacing  $\Delta s$  sufficient to resolve regions of high curvature concentration.

#### 4.11. Composite GCS construction for $G^2$ -continuous transitions

The Single GC procedure above yields a curve with  $\kappa$  varying monotonically from  $\kappa_{min} = \kappa_{max} e^{-\beta}$  to  $\kappa_{max}$ . When the design problem requires a  $G^2$ -continuous tangent-to-tangent transition with  $\kappa(0) = \kappa(L) = 0$ , Mode B of Definition 1 (Composite GCS) is constructed as a symmetric extension of the Single GC procedure.

Let  $\delta_{arc} \in [0, \Delta\theta]$  be a designer-chosen central-arc deflection contribution; the remaining deflection  $\delta_{sp} = \Delta\theta - \delta_{arc}$  is divided equally between two Single-GC half-segments. The half-segment shape is generated by the GC law and shifted so that  $\kappa$  vanishes at the tangent-facing boundary:



$$\hat{\kappa}_{half}(\hat{s}) = \exp(\beta \hat{\theta}(\hat{s})) - \exp(\beta \hat{\theta}(0)), \quad \hat{s} \in [0,1]. \quad (11)$$

then rescaled so that the peak reaches  $\kappa_{max}$ . The half-segment length is  $L_{half} = (\delta_{SP} / 2) / (\kappa_{max} \cdot \int_0^1 \hat{\kappa}_{half} f(\hat{s}) d\hat{s})$ . If  $\delta_{arc} > 0$ , a central circular arc of length  $L_{arc} = \delta_{arc} / \kappa_{max}$  is inserted between the two half-spirals. The total length is  $L = 2 L_{half} + \delta_{arc}$ , with the exit half mirroring the entry. By construction  $\kappa(0) = \kappa(L) = 0$ , and the PCHIP boundary-value duplication ensures  $\hat{\theta}'(0) \approx 0$  and  $\hat{\theta}'(1) \approx 0$  in the discrete numerical implementation, so  $\kappa'(0) \approx 0$  and  $\kappa'(L) \approx 0$  as well. The composite therefore satisfies  $G^2$  continuity at both tangent–curve interfaces.

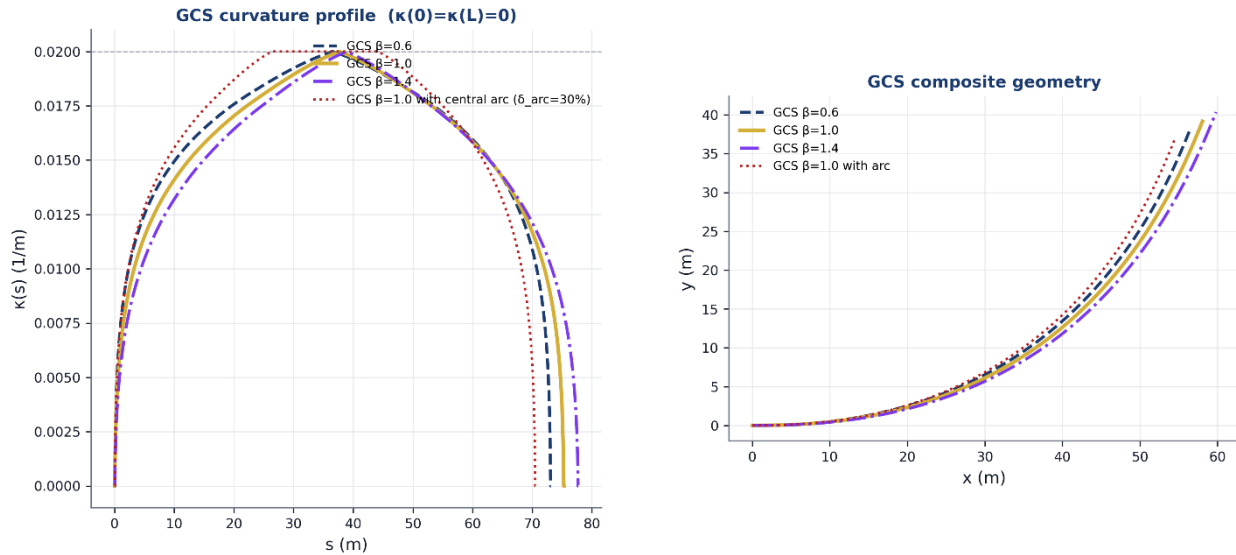


Figure 10. Composite GCS construction: curvature profile (left) satisfies  $\kappa(0) = \kappa(L) = 0$ ; the spatial footprint (right) is consistently more compact than the SCS hybrid reference for the same  $\Delta\theta$ .

## 5. Empirical Example and Engineering Analysis

This section evaluates the proposed framework on three representative design problems: two moderate-deflection cases ( $\Delta\theta = 68^\circ$  and  $90^\circ$ ) and a four-leaf cloverleaf interchange ( $\Delta\theta = 270^\circ$  per loop). For each case, geometric performance is reported alongside vehicle-dynamics indicators, side-friction demand, and constructability considerations under the AASHTO Green Book [1] framework. European national standards (RAS-L Germany, DMRB United Kingdom, ICTAAL France) adopt comparable formulations.

### 5.1. Design scenarios and standard-based parameters

Under the AASHTO equilibrium relation  $R_{min} = \frac{v^2}{127(e+f_{max})}$ , Table 5.1 lists representative design-speed scenarios. The reference curvature bound  $\kappa_{max} = 0.02 \text{ m}^{-1}$  ( $R_{min} = 50 \text{ m}$ ) used in this study corresponds to approximately  $v \approx 40 \text{ km/h}$  under  $e = 0.06$  and is representative of low-speed ramps and urban interchanges.

Table 5.1. AASHTO Green Book [1] design parameters at  $e = 0.06$ .

| $v$ (km/h) | $f_{max}$ (AASHTO) | $R_{min}$ (m) | $\kappa_{max}$ (1/m) |
|------------|--------------------|---------------|----------------------|
| 30         | 0.21               | 26.2          | 0.0381               |
| 40         | 0.18               | 52.5          | 0.0191               |
| 50         | 0.16               | 89.5          | 0.0112               |
| 60         | 0.15               | 135.0         | 0.0074               |
| 80         | 0.14               | 252.0         | 0.0040               |
| 100        | 0.12               | 437.4         | 0.0023               |
| 120        | 0.10               | 708.7         | 0.0014               |

### 5.2. Case A — Horizontal curve, $\Delta\theta = 68^\circ$

Table 5.2. Case A geometric indicators ( $\Delta\theta = 68^\circ$ ,  $\kappa_{max} = 0.02 \text{ m}^{-1}$ ).

| Curve type      | Parameter     | $L$ (m) | Hull ( $\text{m}^2$ ) | Bbox ( $\text{m}^2$ ) | max<br>(m) | $y_{mid}$ | Continuity |
|-----------------|---------------|---------|-----------------------|-----------------------|------------|-----------|------------|
| Pure Circular   | —             | 59.34   | 324.6                 | 1449.6                | 8.55       |           | $G^1$      |
| Single Clothoid | —             | 118.68  | 1285.1                | 4371.6                | 17.43      |           | $G^1$      |
| SCS Hybrid      | $L_s = 40$    | 99.34   | 1084.5                | 3787.5                | 18.57      |           | $G^2$      |
| Single GC       | $\beta = 0.6$ | 64.86   | 391.9                 | 1685.8                | 9.48       |           | $G^1$      |
| Single GC       | $\beta = 1.0$ | 68.53   | 439.8                 | 1848.6                | 10.09      |           | $G^1$      |
| Single GC       | $\beta = 1.4$ | 72.21   | 489.9                 | 2015.3                | 10.69      |           | $G^1$      |



|               |               |       |       |        |       |       |
|---------------|---------------|-------|-------|--------|-------|-------|
| Composite GCS | $\beta = 0.6$ | 73.12 | 530.8 | 2149.6 | 11.66 | $G^2$ |
| Composite GCS | $\beta = 1.0$ | 75.32 | 568.0 | 2274.2 | 12.16 | $G^2$ |
| Composite GCS | $\beta = 1.4$ | 77.73 | 609.9 | 2414.0 | 12.71 | $G^2$ |

Table 5.3. Case A vehicle-dynamics indicators at  $v = 40 \text{ km/h}$ ,  $e = 0.06$ . Peak-jerk values for the GC family are dominated by the steep initial rise of the exponential profile and are localised over a small fraction ( $< 0.5\%$ ) of the curve; RMS jerk is the more representative operational indicator.

| Curve type      | Parameter   | Peak  jerk  ( $\text{m/s}^3$ ) | RMS jerk ( $\text{m/s}^3$ ) | f_dem max |
|-----------------|-------------|--------------------------------|-----------------------------|-----------|
| Pure Circular   | —           | ( $\delta$ at boundary)        | —                           | 0.192     |
| Single Clothoid | —           | 0.231                          | 0.231                       | 0.192     |
| SCS Hybrid      | $L_s=40$    | 0.686                          | 0.560                       | 0.192     |
| Single GC       | $\beta=0.6$ | 17.01 (local)                  | 0.642                       | 0.192     |
| Single GC       | $\beta=1.0$ | 18.42 (local)                  | 0.729                       | 0.192     |
| Single GC       | $\beta=1.4$ | 16.81 (local)                  | 0.707                       | 0.192     |
| Composite GCS   | $\beta=0.6$ | 65.94 (local)                  | 2.509                       | 0.192     |
| Composite GCS   | $\beta=1.0$ | 52.29 (local)                  | 2.090                       | 0.192     |
| Composite GCS   | $\beta=1.4$ | 40.88 (local)                  | 1.736                       | 0.192     |

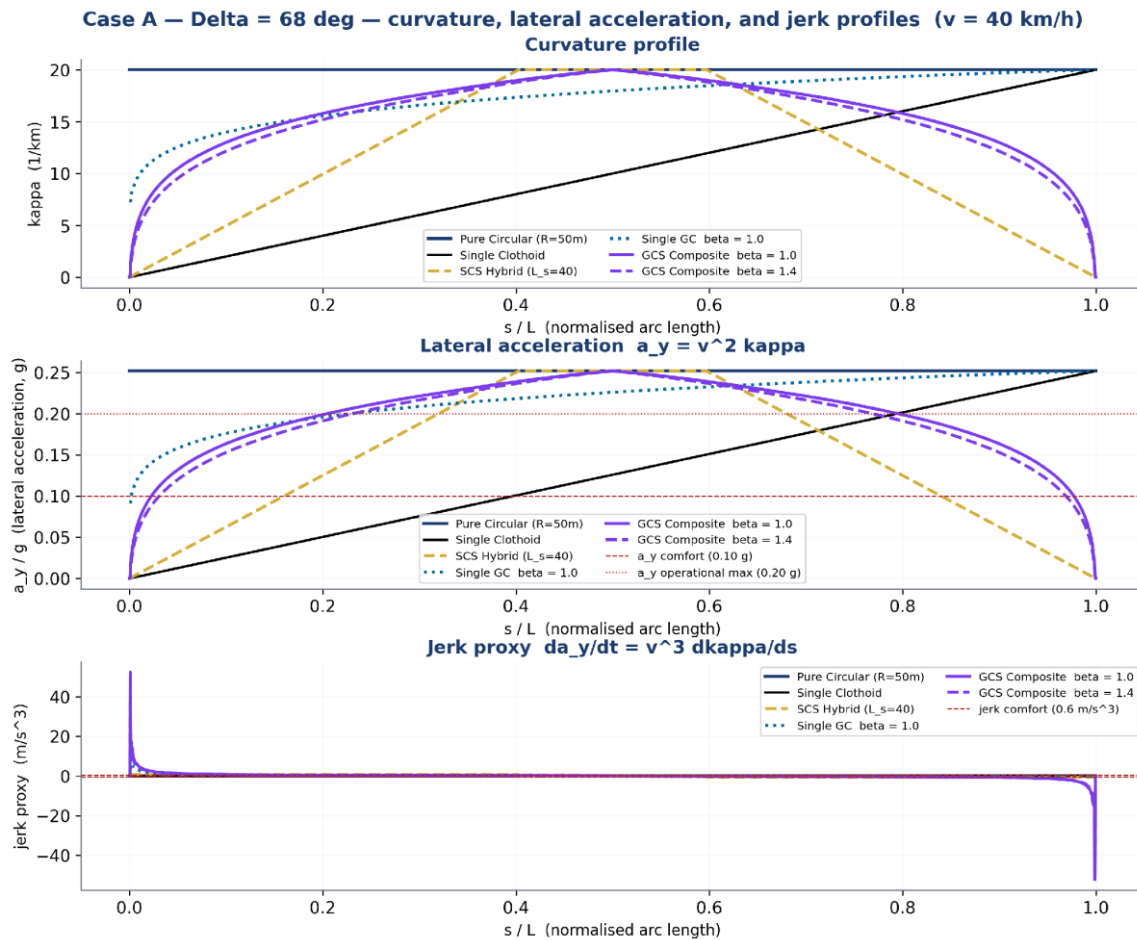


Figure 11. Case A — curvature, lateral acceleration, and jerk profiles at  $v = 40 \text{ km/h}$ .

Observation. Single GC achieves a length reduction of approximately 25–34% relative to SCS hybrid and 39–45% relative to the single clothoid. RMS jerk is moderately higher than the SCS value but remains within operational comfort limits. Composite GCS delivers  $G^2$  continuity at a modestly larger footprint than SCS.

### 5.3. Case B — Horizontal curve, $\Delta\theta = 90^\circ$

Table 5.4. Case B geometric indicators ( $\Delta\theta = 90^\circ$ ,  $\kappa_{max} = 0.02 \text{ m}^{-1}$ ).

| Curve type    | Parameter | L (m) | Hull ( $\text{m}^2$ ) | Bbox ( $\text{m}^2$ ) | max (m) | y_mid | Continuity |
|---------------|-----------|-------|-----------------------|-----------------------|---------|-------|------------|
| Pure Circular | —         | 78.54 | 713.5                 | 2500.0                | 14.64   |       | $G^1$      |



|                 |               |        |        |        |       |       |
|-----------------|---------------|--------|--------|--------|-------|-------|
| Single Clothoid | —             | 157.08 | 2803.6 | 8433.5 | 29.70 | $G^1$ |
| SCS Hybrid      | $L_s = 40$    | 118.54 | 1865.4 | 5072.2 | 27.77 | $G^2$ |
| Single GC       | $\beta = 0.6$ | 85.84  | 860.3  | 2961.1 | 16.23 | $G^1$ |
| Single GC       | $\beta = 1.0$ | 90.71  | 964.6  | 3282.3 | 17.27 | $G^1$ |
| Single GC       | $\beta = 1.4$ | 95.57  | 1073.8 | 3613.4 | 18.29 | $G^1$ |
| Composite GCS   | $\beta = 0.6$ | 96.78  | 1155.2 | 3637.5 | 19.91 | $G^2$ |
| Composite GCS   | $\beta = 1.0$ | 99.69  | 1234.4 | 3838.9 | 20.76 | $G^2$ |
| Composite GCS   | $\beta = 1.4$ | 102.88 | 1323.8 | 4064.2 | 21.69 | $G^2$ |

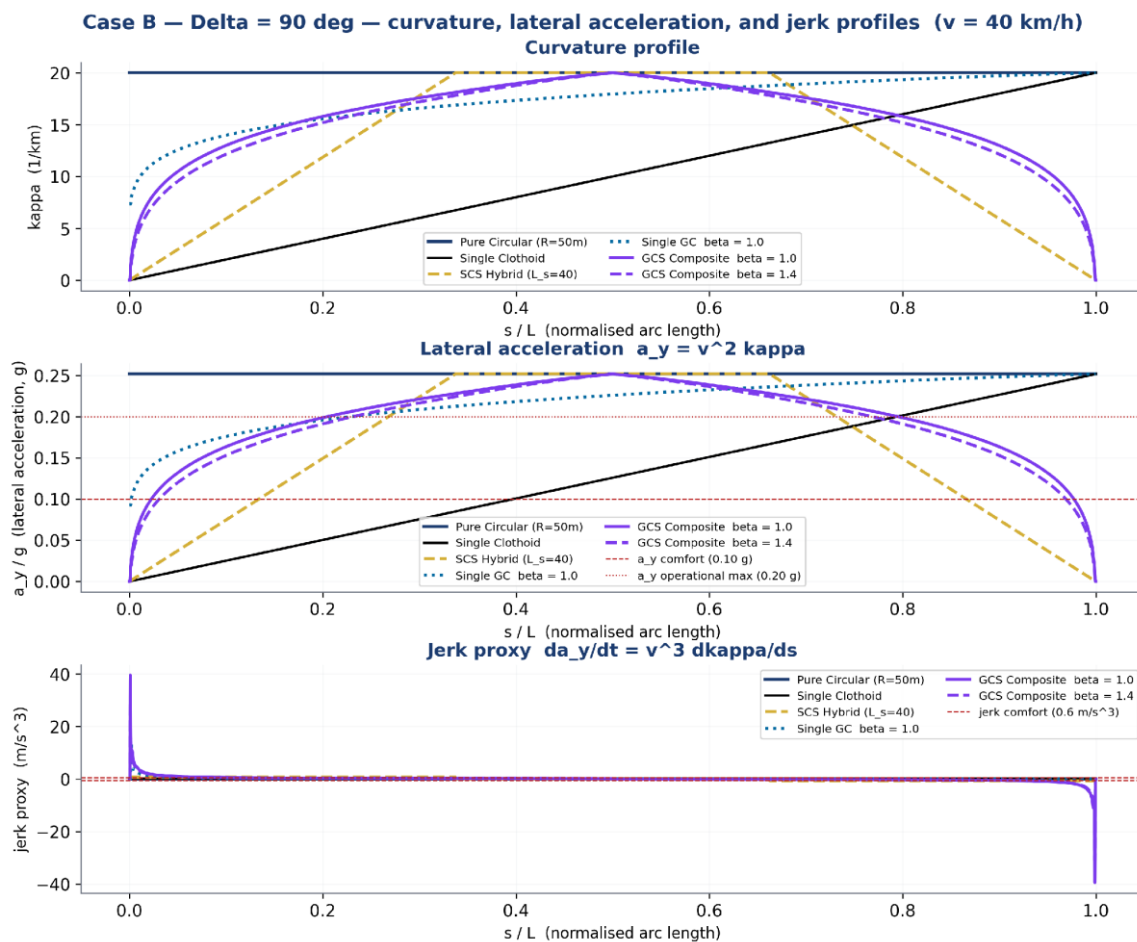


Figure 12. Case B — curvature, lateral acceleration, and jerk profiles at  $v = 40$  km/h.

Observation. At  $\Delta\theta = 90^\circ$ , Single GC reduces curve length by 23–46% and footprint by approximately 35–65% relative to the SCS hybrid and single clothoid. Composite GCS retains  $G^2$  continuity with a footprint comparable to SCS.

#### 5.4. Case C — Cloverleaf interchange, $\Delta\theta = 270^\circ$ per loop

Case C demonstrates a four-leaf cloverleaf interchange in which each loop executes a  $270^\circ$  turn between two perpendicular highways. This is the high-deflection regime where the asymptotic analysis of Section 3 predicts the central-arc segment to dominate the curve length and the spiral contribution to become proportionally smaller. The  $\beta$  range explored here is shifted toward small values (0.1, 0.3, 1.0) because at large deflection the compactness-optimal  $\beta$  regime sits closer to the circular limit.

Table 5.5. Case C — per-loop geometric indicators ( $\Delta\theta = 270^\circ$ ).

| Curve type    | Parameter     | $L_{\text{loop}}$ (m) | Footprint (m $\times$ m) | Continuity |
|---------------|---------------|-----------------------|--------------------------|------------|
| Pure Circular | $R = 50$ m    | 235.62                | 200 $\times$ 200         | $G^1$      |
| SCS Hybrid    | $L_s = 40$    | 275.62                | 203 $\times$ 203         | $G^2$      |
| Single GC     | $\beta = 0.1$ | 239.27                | 204 $\times$ 204         | $G^1$      |
| Single GC     | $\beta = 0.3$ | 246.56                | 211 $\times$ 211         | $G^1$      |
| Single GC     | $\beta = 1.0$ | 272.12                | 237 $\times$ 237         | $G^1$      |
| Composite GCS | $\beta = 1.0$ | 299.08                | 225 $\times$ 225         | $G^2$      |



Table 5.6. Case C vehicle-dynamics indicators at  $v = 40 \text{ km/h}$ . Single GC  $\beta = 0.1$  satisfies the  $0.6 \text{ m/s}^3$  RMS-jerk comfort criterion with significant margin.

| Curve type    | Parameter     | Peak  jerk  ( $\text{m/s}^3$ ) | RMS jerk ( $\text{m/s}^3$ ) | $f_{\text{dem max}}$ |
|---------------|---------------|--------------------------------|-----------------------------|----------------------|
| Pure Circular | $R = 50$      | ( $\delta$ at boundary)        | —                           | 0.192                |
| SCS Hybrid    | $L_s = 40$    | 0.686                          | 0.560                       | 0.192                |
| Single GC     | $\beta = 0.1$ | 1.229                          | 0.044                       | 0.192                |
| Single GC     | $\beta = 0.3$ | 2.965                          | 0.109                       | 0.192                |
| Single GC     | $\beta = 1.0$ | 4.640                          | 0.184                       | 0.192                |
| Composite GCS | $\beta = 1.0$ | 13.17 (local)                  | 0.526                       | 0.192                |

Case C — Delta = 270 deg (cloverleaf) — curvature, lateral acceleration, and jerk profiles ( $v = 40 \text{ km/h}$ )

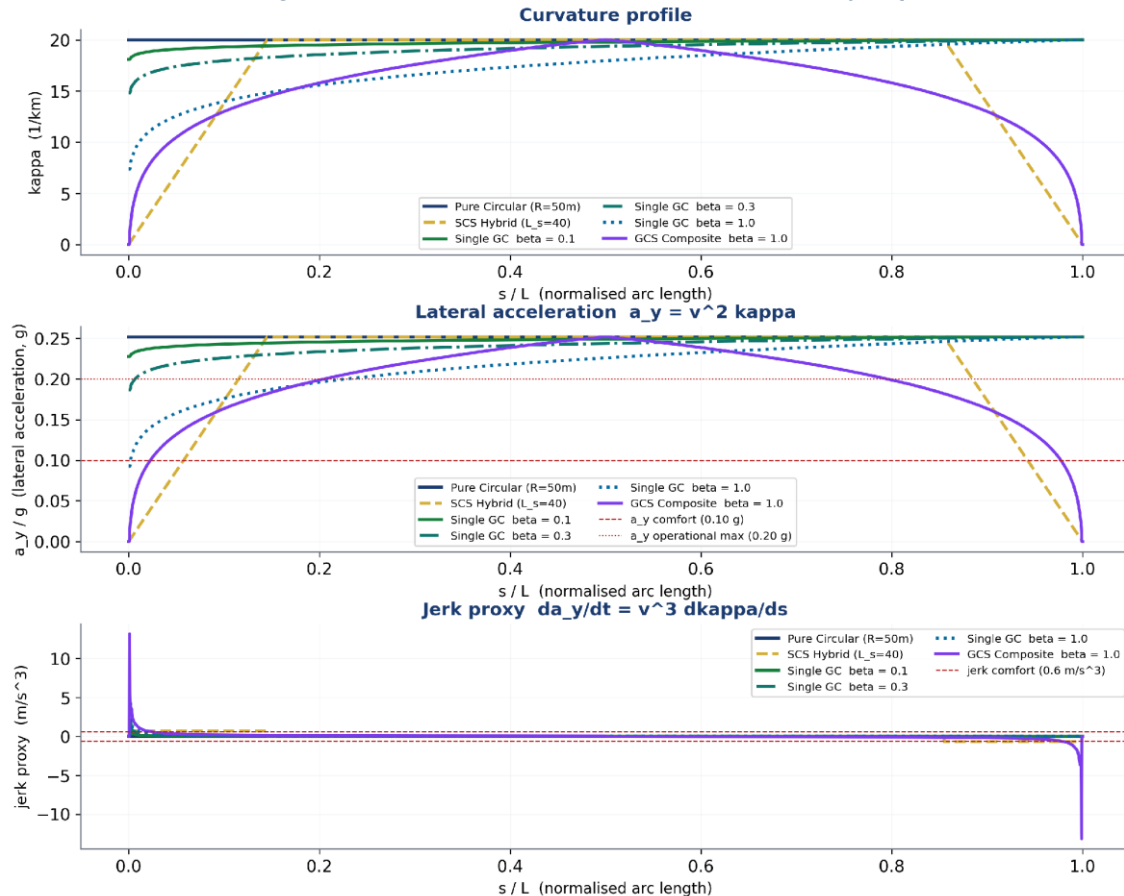


Figure 13. Case C — curvature, lateral acceleration, and jerk profiles at  $v = 40 \text{ km/h}$ .

Observation. At  $\Delta\theta = 270^\circ$ , the small- $\beta$  regime of Single GC offers a particularly attractive trade-off: Single GC  $\beta = 0.1$  produces a footprint only 2% larger than the pure circular optimum and satisfies the RMS-jerk comfort criterion with significant margin. This regime combines compactness and smoothness in a way that the classical clothoid-based designs cannot match.

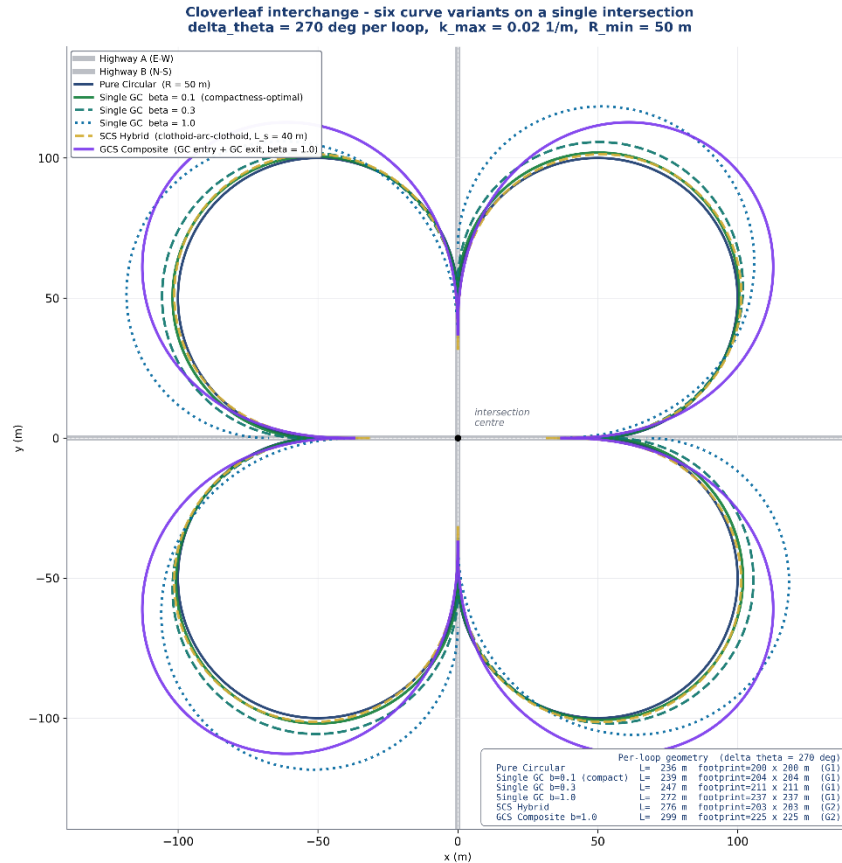


Figure 14. Illustration of cloverleaf Interchange with six curve variants

## 5.5. Engineering integration

**Side-friction demand.** The peak side-friction demand  $f_{demand,max} = \frac{v^2 \kappa_{max}}{g} - e \approx 0.192$  at  $v = 40$  km/h,  $e = 0.06$  slightly exceeds the AASHTO comfort-design limit  $f_{max} = 0.18$ . Two standard remediations bring all designs fully within the standard envelope:  $e = 0.08$  yields  $f_{demand,max} \approx 0.172$ , and  $v = 35$  km/h yields  $f_{demand,max} \approx 0.149$ . Both options are routine in cloverleaf and urban-ramp design. The relative ranking among curve families is unaffected by the choice of remediation.

**Superelevation balance.** At  $v = 40$  km/h and  $\kappa_{max}$ ,  $e_{balance}$  reaches 0.252 (above standard pavement-superelevation limits) so practical design relies on the combination of  $e$  and  $f_{demand}$ . The GC framework does not change this physical balance but does change the spatial profile of  $\kappa(s)$ , with implications for superelevation-runoff length and pavement-edge geometry.

**Constructability.** The number of geometric segments is 1 for the Single GC, 2–3 for the Composite GCS (depending on whether a central arc is included), and 3 for the SCS hybrid. Single GC offers minimal-segment construction among curvature-continuous families, simplifying field set-out and CAD/BIM integration. The smoothness order at internal junctions is operationally relevant: SCS is  $G^1$  at spiral-arc junctions ( $\kappa$  continuous,  $d\kappa/ds$  jumps),



whereas Composite GCS is  $G^2$  across the apex and at all internal junctions.

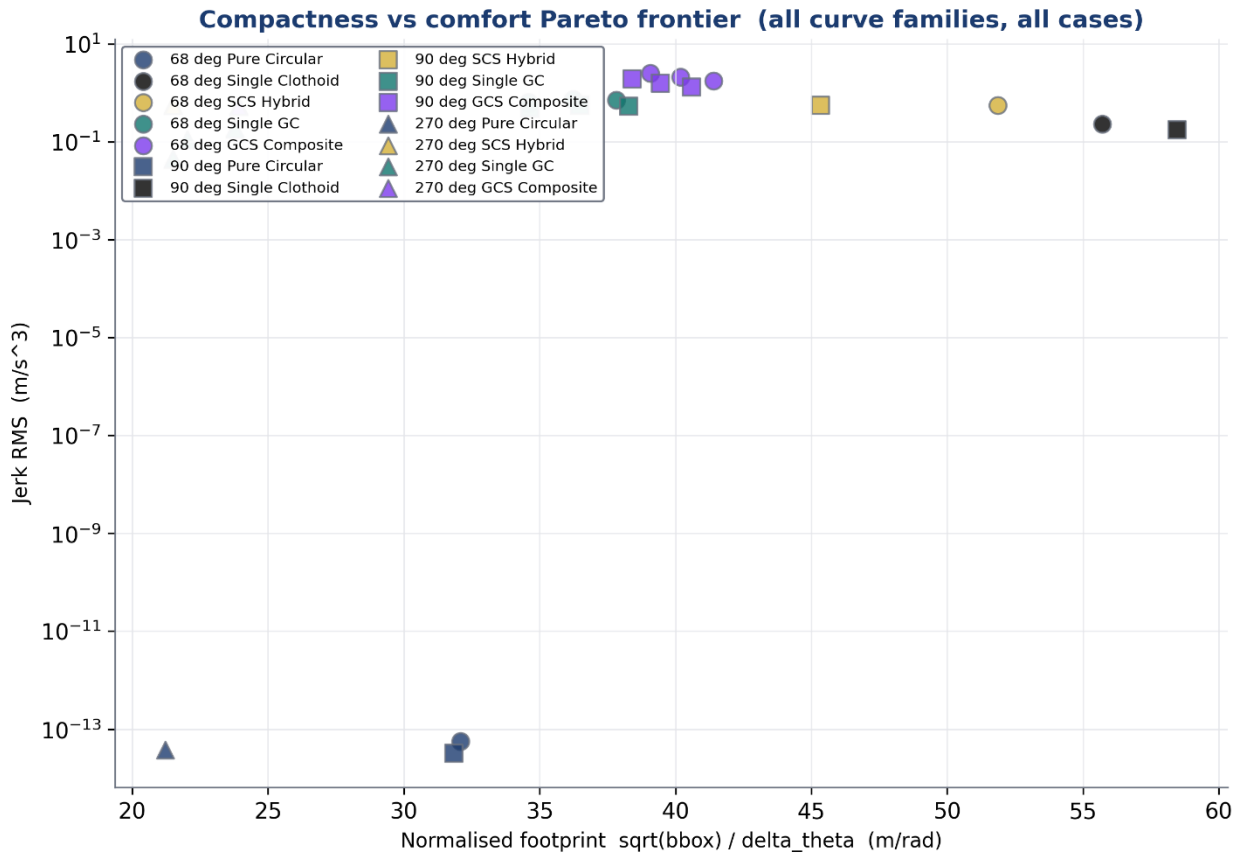


Figure 15. Compactness–comfort Pareto frontier across all curve families and case studies. Single GC family forms the small-footprint branch; clothoid family forms the low-jerk branch

## 6. Discussion

The empirical evidence of Section 5 demonstrates that the Golden Curvature framework operates as a complement to, rather than a replacement for, the classical clothoid-based horizontal-alignment standards. This section consolidates the engineering positioning, maps the operational trade-offs, and identifies limitations and future-work directions.

### 6.1. Positioning within established design practice

AASHTO Green Book [1] and equivalent European national standards prescribe minimum horizontal radii through  $R_{min} = \frac{v^2}{127(e+f_{max})}$  and require curvature-continuous transitions wherever lateral-acceleration build-up is a comfort or safety concern. The Golden Curvature framework integrates into this practice in three complementary ways. First, GC respects the same hard constraints ( $\kappa_{max}$ ,  $\Delta\theta$ ,  $v$ ,  $e$ ,  $f_{max}$ ) and does not require any modification of the standard design-input set;  $\beta$  and  $N$  operate within the existing envelope as additional tuning variables. Second, GC contains the classical curve families as analytically identifiable limit cases (circular at  $\beta \rightarrow 0$ , clothoid at  $\beta = \log F_N$ , logarithmic spiral at  $\beta = 1$ ,  $N \rightarrow \infty$ ) and may be treated as a continuous parametric superset of these paradigms. Third, the two construction modes (Single GC and Composite GCS) correspond to two distinct operational scenarios, Single GC for compactness-oriented  $G^1$  applications, Composite GCS as the direct analogue of SCS for  $G^2$ -continuous tangent-to-tangent transitions.

### 6.2. The compactness–comfort trade-off

The empirical analyses establish that the GC framework does not deliver Pareto improvements over the clothoid family in every metric. The single clothoid is variationally optimal for the jerk-energy functional  $\int (d\kappa/ds)^2 ds$  under fixed  $\kappa(0) = 0$ ,  $\kappa(L) = \kappa_{max}$  and prescribed  $\Delta\theta$ ; this is a classical result and cannot be outperformed. What the GC framework delivers is a controllable trade-off between geometric compactness and curvature-evolution smoothness, governed by  $\beta$ . Two operational regimes emerge: small  $\beta$  ( $\leq 0.3$ ), in which curvature varies gently, the design approaches the pure-circular geometric optimum, and comfort indicators remain well within standard envelopes (appropriate for high-deflection applications such as cloverleaf ramps and hairpin turns; and moderate  $\beta$  ( $0.6 - 1.4$ ), in which curvature concentrates progressively toward the apex with measurable savings in length and footprint relative to clothoid-based alternative) appropriate for low-to-medium-speed applications where geometric flexibility is needed.



### 6.3. Selection guidelines

Table 6.1. Curve-family selection guidelines under AASHTO design practice.

| Design context                                                                 | Recommended curve family                               | Rationale                                                                                                           |
|--------------------------------------------------------------------------------|--------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| High-speed mainline ( $v \geq 60$ km/h)                                        | Single Clothoid / SCS Hybrid                           | Variational jerk-optimality of clothoid family dominates at high speed.                                             |
| Moderate-speed urban ( $\Delta\theta \leq 90^\circ$ , $v \approx 30$ –50 km/h) | Single GC ( $\beta \approx 0.6$ –1.0) or Composite GCS | Compactness savings of 25–40% relative to SCS at modest jerk premium; mode choice depends on $G^1/G^2$ requirement. |
| Cloverleaf / hairpin loop ( $\Delta\theta \geq 180^\circ$ )                    | Single GC ( $\beta \approx 0.1$ –0.3)                  | Approaches pure-circular optimum with monotone curvature build-up and standard-compliant RMS jerk.                  |
| Tight-space retrofit, parking ramps, taxiways                                  | Single GC (small $\beta$ )                             | Geometric compactness is the binding constraint.                                                                    |
| Autonomous-vehicle path-planning baseline                                      | Composite GCS or Single GC (small $\beta$ )            | Higher-order curvature smoothness simplifies MPC cost-function formulation.                                         |

### 6.4. Limitations

The empirical evidence rests on a number of modelling assumptions. (i) Constant-speed analysis: lateral-acceleration and jerk profiles assume constant vehicle speed; coupling with speed-varying driver models is open. (ii) Closed-form vehicle dynamics: the vehicle is modelled as a point mass; full bicycle-model or multi-body simulation is identified as future work. (iii) Single curvature bound: the analysis uses  $\kappa_{max} = 0.02 \text{ m}^{-1}$  across cases; a multi-speed sensitivity sweep is feasible within the same framework. (iv) Symmetric design: Composite GCS uses identical entry and exit  $\beta$ ; asymmetric designs ( $\beta_{entry} \neq \beta_{exit}$ ) are consistent with the framework but not explored here. (v) No field validation: the analyses are numerical; instrumented field validation is identified as a high-priority future direction. (vi) Peak-jerk localisation: peak-jerk values for the GC family are large but spatially localised at the boundary ( $< 0.5\%$  of curve length); RMS jerk is the more representative operational indicator. Smoother choices of the scaling map  $\theta$  (e.g. higher-order analytic interpolation) may reduce the peak-jerk magnitude and are a target of future work.

### 6.5. Future work

Immediate priority: full vehicle-dynamics simulation of Single GC and Composite GCS under realistic speed-varying and steering-input profiles; systematic exploration of asymmetric  $\beta$ -pair designs; investigation of higher-order scaling maps  $\theta$  to mitigate boundary peak-jerk excursions. Medium-term: field validation through surveyed implementation of a Composite GCS alignment in a low-speed test-bed; extension to combined horizontal-and-vertical alignment with three-dimensional curvature continuity; integration with model-predictive control architectures for autonomous-vehicle path planning. Long-term: adaptive  $\beta$ -profile design with  $\beta$  varying along the alignment as a function of design-speed profile; multi-objective shape optimisation incorporating earthwork volume, environmental impact, and energy consumption.

## 7. Conclusion

This study introduced the Golden Curvature (GC) framework, a Fibonacci-anchored, parametric, curvature-continuous family for horizontal road alignment design. The framework is characterised by a single dimensionless intensity parameter  $\beta$  and admits two construction modes: Single GC (monotone single-segment,  $G^1$ ) and Composite GCS (symmetric two-segment with optional central arc,  $G^2$ ). Asymptotic analysis demonstrated that the framework recovers the circular arc ( $\beta \rightarrow 0$ ), the clothoid ( $\beta = \log F_N$ ), and the logarithmic spiral ( $\beta = 1, N \rightarrow \infty$ ) as analytically identifiable limit cases, providing a unified parametric superset of the classical curve families. Three case studies ( $\Delta\theta = 68^\circ$ ,  $\Delta\theta = 90^\circ$ , and a  $\Delta\theta = 270^\circ$  cloverleaf interchange) were evaluated under AASHTO Green Book [1] design-speed scenarios, with complementary vehicle-dynamics, side-friction, superelevation, and constructability analyses. The Single GC mode yielded 25–45% shorter curve lengths and up to 65% smaller spatial footprints than the SCS hybrid at moderate deflections; at high deflections the small- $\beta$  regime approached the pure-circular optimum to within 2% while satisfying the operational comfort criteria. Composite GCS provided  $G^2$ -continuous tangent transitions with a footprint comparable to the SCS hybrid. All designs remained within the AASHTO side-friction envelope after routine remediation through superelevation or design-speed adjustment. The framework is positioned as a complement (not a replacement) to the classical clothoid family. It is most valuable in design contexts where the clothoid-based approach is either over- or under-prescribed: too long when geometric compactness is binding, or too rigid when smoothness order beyond  $G^2$  is operationally meaningful. The framework is fully compatible with established standards-based design workflows, requires no modification of the standard input set, and exposes two new tuning variables ( $\beta$  and  $N$ ) within the existing constraint envelope. Future work will address vehicle-dynamics simulation, field validation, asymmetric designs, three-dimensional extension, and integration with autonomous-vehicle path-planning architectures.



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## Conflicts of Interest

The authors declare that there is no conflict of interest.

## Data Availability Statement

All source observations are given as supplementary material.

## Institutional Review Board Statement

This study does not involve human participants, animal subjects or personal data, and therefore requires no ethics committee approval.

## CRedit Author Statement

Abdullah KARAAĞAÇ: Conceptualization; Methodology; Software; Validation; Formal analysis; Investigation; Data curation; Writing – original draft; Writing – review & editing; Visualization.

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